Estimation of the Capacity of Multipath Infrared Channels

Jeffrey B. Carruthers
Department of Electrical and Computer Engineering
Boston University
jbc@bu.edu

Sachin Padma
Department of Electrical and Computer Engineering
Boston University
sachinp@bu.edu

ABSTRACT
We present a method for estimating the channel capacity of multipath intensity-modulation infrared links. The capacity under both peak-power constraint and peak and average-power constraint is evaluated. No other assumptions about the structure of the input signal are imposed, so the result applies for pulse-amplitude modulation, pulse-position modulation, or any other signal format. The capacity of any arbitrary impulse response can be calculated as a function of peak and average signal-to-noise ratio in the presence of additive white Gaussian noise. We verify that in both cases the capacity is achieved by a unique input distribution taking on a finite number of mass points. As an example application of the method, we evaluate the capacity of a family of multipath channels based on a ceiling-bounce model. The channels are examined for various values of delay spread at peak and average SNR values ranging from 0 dB to 8 dB. The capacity of a typical ceiling bounce model channel with a 10 ns delay spread is found to range from 8 Mb/s to 28 Mb/s with a peak SNR limit of 6 dB, depending on the strictness of the average power constraint.

KEY WORDS
Wireless Infrared Channels, Information Capacity, Multipath

1. Introduction
High-quality wireless access to information, networks, and computing resources by users of portable computing and communication devices is driving recent activity in indoor infrared communication [1, 2]. High-quality access is achieved via links with low delay, high data rates, and reliable performance, and accurate characterization of the channel is essential to understanding the performance limits and design issues for wireless infrared links.

Portable terminals using infrared links are subject to severe limitations on power consumption. Transmitter power may also be limited by concerns of eye safety. This imposes a constraint on the peak power and average power of the transmitter. In practical infrared systems, the peak power constraint arises from considerations of available source (laser or LED) power. Concerns of eye safety and battery life generally determine the average-power constraint. Although both the constraints are equally important, the latter is much stricter and difficult to relax. The peak-power constraint can be overcome by using a more powerful laser.

We consider the capacity of intensity-modulated direct detection(IM/DD) infrared wireless links. In [3] an algorithm to estimate the channel capacity for peak- and average-limited channels with white Gaussian noise is presented. All the results have been developed for channels in the absence of inter-symbol interference. In [4] the authors describe a novel algorithm to estimate the channel capacity of infrared channels in presence of inter-symbol interference for pulse amplitude modulated signals, but they restrict their attention to independent input symbols.

In the next section, we describe our channel model and the basis for our capacity calculation. In Section 3, we show how the capacity under a peak-power constraint can be determined, then we add the average-power constraint in Section 4. We then apply the method to study the capacity of typical multipath channels in Section 5. Concluding remarks are in Section 6.

2. Channel Model
In IM/DD channels, the desired waveform is modulated onto the instantaneous power of the carrier. The receiver performs direct detection in which a photodiode produces a current proportional to the instantaneous received power.

The channel input waveform $x(t)$ is the instantaneous optical power of the infrared transmitter. The channel output waveform $y(t)$ is instantaneous current of the receiving photo-detector. This current is proportional to the integral of the instantaneous total power at each location over the photo-detector surface. Since there exist multiple paths between transmitter and receiver, the received energy exhibits spatial variation in magnitude and phase. This variation would result in multipath fading if the detector was smaller than wavelength of the signal. However, typical detectors are orders of magnitude larger than the wavelength of the
signal resulting in spatial diversity which prevents multi-path fading.

However, since the transmitted signal propagates along multiple paths of different lengths, the received signal is still subject to multipath distortion. The channel can thus be modeled as a linear system with input power \( x(t) \), output current \( y(t) \), and an impulse response \( h(t) \). The channel impulse response \( h(t) \) is quasi-static as it changes only when the transmitter, receiver or objects in the room are moved by tens of centimeters.

Infrared links generally operate in the presence of visible background light including sunlight, incandescent light, and fluorescent light, which can be modeled as white Gaussian noise independent of signal \( x(t) \). Thus in our channel we model noise \( z(t) \) as additive white Gaussian noise.

In conclusion, our baseband channel can be summarized by

\[
y(t) = x(t) * h(t) + z(t)
\]

where the “∗” symbol denotes convolution. While (1) denotes the system equation of a conventional linear filter channel with additive noise, it differs from conventional electrical systems in many ways. Firstly, since \( x(t) \) represents instantaneous optical power of the transmitter, it is non negative i.e.,

\[
x(t) \geq 0.
\]

Secondly, the average transmitted optical power is given by the average of signal \( x(t) \) rather than the average of \( |x(t)|^2 \) which holds for conventional electrical signal channel where \( x(t) \) denotes amplitude i.e.,

\[
P_{av} = E(x(t)) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t)dt
\]

Therefore the results developed for electrical signals are not applicable directly to wireless infrared channels.

The average transmitted optical energy is given by \( E(x(t)) \). We model \( z(t) \) as zero mean white Gaussian random variable with variance \( \sigma^2_N \) and without loss of generality we normalize the noise standard deviation \( \sigma_N \) to unity. We define signal to noise ratio (SNR) as the ratio of optical power (either peak or average) arriving at the receiver and variance of the noise.

The equivalent discrete time channel can be described as

\[
y_k = x_k + \sum_{l=1}^{\infty} h_l x_{k-l} + z_k \quad -\infty < k < \infty
\]

where \( x_k \) and \( z_k \) are the discretized versions of input signal and noise signal obtained by sampling \( x(t) \) and \( z(t) \) every \( T_s \) seconds. Also \( h_1 \) is the equivalent discrete time impulse response. To calculate \( h_1 \) we assume a transmit pulse \( g(t) \) of time period \( T_s \), which is the symbol period of input \( x(t) \). We then convolve \( g(t) \) with channel impulse response \( h(t) \) and apply a transmit-matched filter at the receiver \( g(t) \) and sample. Thus, we have

\[
c(t) = g(t) * h(t) * g(t)
\]

Throughout, we use

\[
g(t) = \begin{cases} \frac{1}{T_s} & |t| < \frac{1}{2T_s} \\ 0 & \text{elsewhere} \end{cases}
\]

An alternative vector representation of above input-output relationship is

\[
y^N = H^N x^N + z^N
\]

where \( y^N = (y_0, y_1, \ldots, y_{N-1}) \), \( x^N = (x_0, x_1, \ldots, x_{N-1}) \) and \( z^N = (z_0, z_1, \ldots, z_{N-1}) \) are output, input and noise column vectors and \( H^N \) is channel coefficient matrix given by

\[
H^N = \begin{bmatrix} h_0 & 0 & 0 & 0 & \ldots \\ h_1 & h_0 & 0 & 0 & \ldots \\ h_2 & h_1 & h_0 & 0 & \ldots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ h_{N-1} & h_{N-2} & \ldots & h_1 & h_0 \end{bmatrix}
\]

The capacity of the above channel in bits/second is given by

\[
C(T_s) = \frac{1}{T_s} \lim_{N \to \infty} \frac{1}{N} \int I(y^N; z^N)
\]

where \( C(T_s) \) denotes the capacity of the discrete time channel obtained by sampling the continuous time channel every \( T_s \) seconds. It can be shown that the information capacity of the channel is given by

\[
C = \lim_{T_s \to 0} C(T_s).
\]

Therefore, we consider longer sequences \( (N \to \infty) \) and short symbol periods \( (T_s \to 0) \) to approximate the capacity of the infrared link having a continuous-time channel impulse response.

### 3. Capacity under Peak-Power Constraint

The basic problem of finding the capacity of wireless infrared link impaired by inter-symbol interference under peak power constraint can be stated as:

For an input random variable \( x^N = (x_0, x_1, \ldots, x_{N-1}) \) and output random variable \( y^N = (y_0, y_1, \ldots, y_{N-1}) \), find the optimal probability distribution function of \( x^N \) such that mutual information between input and output is maximized subject to peak power constraint, i.e., find

\[
\max_{p^N_2(x^N)} I(x^N; y^N)
\]

such that \( 0 \leq x_i \leq A \) for \( i = 0 \) to \( N-1 \) where \( A \) is the peak power constraint. The peak SNR is thus \( 10 \log_{10} A \) dB.
3.1 Optimization Problem

We denote the joint cumulative distribution function of \( x^N \) by \( F^N(x) \) and the space of \( F^N(x) \) is denoted by \( \mathcal{F}_A \). The N-dimensional joint probability density functions of \( z^N \) and \( y^N \) are denoted by \( p^N_{z}(z^N) \) and \( p^N_{y}(y^N) \) respectively.

The channel capacity subject to peak power constraint \((0 \leq x^N \leq A)\) is identical to the channel capacity subject to \( -\frac{A}{2} \leq x^N \leq \frac{A}{2} \) because the shift in the d.c. level does not make any difference to the mutual information calculations. The problem we are studying is in N-dimensional space and we use the results developed in [3] as they are equally applicable in this case. We apply the techniques in [3] and find that the capacity is achieved by a unique distribution function in \( \mathcal{F}_A \), denoted by \( F^N_0 \). The set of mass points of \( F^N_0 \), denoted by \( M^N_0 \), must satisfy

\[
\begin{align*}
\forall x^N \in [0, A]^N, & \quad i(x^N; F^N_0) \leq I(F^N_0), \\
\forall x^N \in M^N_0, & \quad i(x^N; F^N_0) = I(F^N_0),
\end{align*}
\]

where \( i(x^N; F^N) \) is the marginal density function of the mutual information between \( x^N \) and \( y^N \) for the distribution \( F^N \). Finally, since \( M^N_0 \subset [0, A]^N \), it can be shown that \( M^N_0 \) is a set of finite size.

Therefore the problem of estimating the capacity turns out to be a simple constrained optimization problem where we maximize the mutual information which is a function of a finite dimensional vector whose components are the mass point positions and mass point values of the input distribution. The difficulty is that we do not know the size of \( M^N_0 \). There is no explicit formula for calculating the size of \( M^N_0 \). To determine \( |M^N_0| \), one tries different values of \( |M^N_0| \) to find the one that satisfies the optimality conditions in (8) and (9).

The mutual information functional is evaluated as

\[
I(x^N; y^N) = H(y_{N-1}, \ldots, y_0) - H(y_{N-1}, \ldots, y_0|x_{N-1}, \ldots, x_0) = H(y_{N-1}, \ldots, y_0|x_{N-1}, \ldots, x_0) - H(y_{N-1}|y_{N-2}, \ldots, y_0, x_{N-1}, \ldots, x_0) - \ldots
\]

where the latter \( N \) conditional entropy terms are equal to the noise entropy which is a constant and is equal to \( D = \log 2\pi e \). Therefore

\[
I(x^N; y^N) = H(y_{N-1}, y_{N-2}, \ldots, y_0) - ND
\]

In (10) above, the entropy term of \( N \) output variables is calculated as

\[
H(y_{N-1}, y_{N-2}, \ldots, y_0) = \int_0^A \int_0^A p(y_{N-1}, \ldots, y_0) \log p(y_{N-1}, \ldots, y_0) dy_0 \ldots dy_{N-1}
\]

where the integration is repeated \( N \) times. The probability distribution of the output variables \( p(y_{N-1}, y_{N-2}, \ldots, y_0) \) is obtained from the input and noise probability distributions as shown at the top of the next page.

Hence the conditional probability density function of the output given the input is simply the probability density function of the Gaussian noise.

The necessary and sufficient conditions in (8) and (9) provide a test to determine whether the actual \( |M^N_0| \) is equal to the assumed \( |M^N_0| \). When the optimality test fails, we increase \( |M^N_0| \) by one in each dimension and the whole procedure is repeated. We assume a starting point for \( x^N \) obtained by considering for each \( x_i \) mass point located at extremes i.e., one at 0 and one at \( A \) with equal probability i.e., we begin with \( |M^N_0| = 2^N \) and then perform a constrained search for the distribution of \( x^N \) which maximizes the mutual information function.

4. Capacity under Peak- and Average-Power Constraint

In this section we explore the problem of estimating the channel capacity given both a normalized average-power constraint and a normalized peak-power constraint. The problem can be stated as

\[
\max_{p^N_{x}(x)} I(x^N; y^N)
\]

such that \( 0 \leq x_i \leq A \) for all \( i = 0 \) to \( N - 1 \) and \( \sum_{i=0}^{N-1} E(x_i) \leq \rho N \) where \( \rho \) is the average power constraint and \( A \) as previously stated the peak-power constraint.

We follow the development in [5] to incorporate average power constraint in analysis of the previous section. Now, the set of mass points \( M^N_0 \) of the unique capacity-achieving distribution \( F^N_0 \) satisfy

\[
i(x^N; F^N_0) \leq I(F^N_0) + \lambda(x^N - \rho), \forall x^N \in [0, A]^N, \quad (11)
\]

and

\[
i(x^N; F^N_0) = I(F^N_0) + \lambda(x^N - \rho), \forall x^N \in M^N_0, \quad (12)
\]

where \( \lambda \) is the Lagrangian multiplier. Since \( M^N_0 \subset [0, A]^N \), it can be shown that \( M^N_0 \) is a set of finite size.

The algorithm is the same as developed for peak-power constraint except that there is an additional non-linear constraint on the average power of input distribution specified as \( \sum_{i=0}^{N-1} E(x_i) \leq \rho N \).

5. Results

We now use an MATLAB-based implementation of the preceding methods to evaluate capacities for a typical family of impulse responses based on the ceiling bounce model of [6]. The impulse response of such a channel is given by the expression

\[
h(t) = \frac{H(0)6a^6}{(t + a)^7} u(t)
\]
\[ p(y_{N-1}, y_{N-2}, \ldots, y_0) = \sum_{x_0} \sum_{x_1} \ldots \sum_{x_{N-1}} p^N_{x}(x_0, x_1, \ldots x_{N-1}) p(y_0, y_1, \ldots y_{N-1}|x_0, x_1, \ldots x_{N-1}) \]
\[ = \sum_{x_0} \ldots \sum_{x_{N-1}} p^N_{x}(x_0, \ldots x_{N-1}) p^N_{z}(y_{N-1} - h_0 x_{N-1} - \ldots - h_{N-1} x_0) \]
\[ p^2_{z} (y_{N-2} - h_0 x_{N-2} - \ldots - h_{N-2} x_0) \ldots p^N_{z}(y_0 - h_0 x_0) \]

where the channel constant \( a = 2D/c \) and \( D \) is the height of the ceiling above the transmitter and the receiver and \( c \) is speed of light. We choose \( H(0) = 1 \) so that the channel has a path loss of 0 dB. The delay spread can be varied using the parameter \( a \). These channels are typical of multipath structure experienced in many diffuse infrared channels.

We evaluated the capacity under both peak-power and peak- and average-power constraints. Our strategy was to start with two (i.e., \( N = 2 \)) symbols and increase the number of symbols and evaluate the total capacity as \( N \) increases. As \( N \) increases the plot of total capacity versus \( N \) becomes linear. The slope of this straight line gives us the capacity per symbol. All the channel capacities obtained by this algorithm are lower bounds on the actual channel capacity.

### 5.1 Capacity under Peak-power Constraint

In Fig. 1, we show the estimated capacity for two symbol times \( T_s \). The information capacity in bits per second is obtained by multiplying the capacity in bits per symbol with the symbol rate (\( 1/T_s \)). The capacity per symbol for symbol period of 20 ns is less than that of 30 ns because the ISI is greater when we use short symbol periods. However, the overall capacity is greater as one can transmit more symbols. We expect that as \( T_s \) is shortened, additional plots would quickly converge to the true capacity.

In Fig. 2, we examine the effect of delay spread on the channel capacity. Here, the symbol period is 30 ns and the delay spread is varied between 0 ns (no ISI) to 10 ns. The capacity penalty due to the multipath spread is significant, with the largest relative decline in capacity experienced in the low SNR case. The ISI penalty is shown explicitly in Fig. 3, which compares the capacity of the 10 ns rms delay spread ceiling bounce impulse response and an ideal (non-distorting) channel.

The issue of the symbol period required to obtain an accurate estimate of the channel capacity is examined in Fig. 4. It shows the estimated capacity of the ideal channel for different peak SNR values as the symbol period is varied. The graph shows that as expected, the capacity estimate approaches an asymptote as \( T_s \) approaches 0. Further, the larger the capacity of the channel, the smaller the value of \( T_s \) one needs to consider for accurate results.

![Figure 1](image1.png)  
**Figure 1.** Capacity versus normalized peak-power for the ceiling-bounce impulse response with delay spread of 10 ns.

![Figure 2](image2.png)  
**Figure 2.** Capacity in bits/symbol versus delay spread. The capacity of ceiling bounce model for normalized peak-power values of 2, 4, 6 dB; \( T_s = 30 \) ns.
Figure 3. Capacity in bits/symbol versus normalized peak-power plotted for two channels, a ceiling-bounce model channel with delay spread of 10 ns and symbol period $T_s=30$ ns and a channel with no ISI.

Figure 4. Capacity versus symbol period for a channel with no ISI.

Figure 5. Capacity in bits/symbol versus normalized average-power constraint. The peak-power constraint is varied as $A = \rho \times \text{const}$ where $\text{const} = 2, 4, 8, 16$. The channel rms delay spread is 10 ns and $T_s=30$ ns.

Figure 6. Capacity in bits/symbol versus normalized peak-power constraint. The channel rms delay spread is 10 ns and $T_s=30$ ns.
5.2 Capacity under Peak- and Average-Power Constraints

Fig. 5 shows the channel capacity as a function of the normalized average-power constraint at several particular values of peak-to-average-power ratios. The capacity given a peak-to-average-power ratio of 2 is exactly same as the capacity with only a peak-power constraint. It is apparent that given a fixed average-power constraint, systems which can tolerate a higher peak-power constraint have higher capacities. In Fig. 6, the channel capacity is replotted against the peak-power constraint for various average-power power values.

6. Conclusions

We have described, implemented, and tested a method to estimate the IM/DD channel capacity under peak-power and peak- and average-power constraints to account for multipath-induced inter-symbol interference.

We estimated the channel capacity of wireless infrared links with a ceiling bounce channel impulse response model. This demonstrated a decrease in the channel capacity with increased multipath spread. We also showed that both the peak-power and average-power constraints can have a significant impact on the achievable capacity. The capacity of a typical ceiling bounce model channel with a 10 ns delay spread is found to range from 8 Mb/s to 28 Mb/s with a peak SNR limit of 6 dB, depending on the strictness of the average power constraint.

The complexity of the algorithm to calculate the entropies and mutual information functional is exponential in the number of symbols considered. Therefore the process of obtaining the optimal distribution and estimating the capacity is a time consuming process as it requires numerous evaluations of these entities. We have evaluated capacities considering symbol sets up to five symbols. If computational facilities allow for larger symbol sets, a much stricter bound on the capacity can be obtained. Furthermore, to closely approximate the capacity of continuous-time channel, smaller symbol times need to be used.

References


