LEVEL SET METHODS FOR DYNAMIC TOMOGRAPHY

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ABSTRACT

In this paper, we propose a novel variational framework for the reconstruction of dynamic objects from sparse and noisy tomographic data. Using an object-based scene model, we developed a general object dynamic model based on a one to one and differentiable mapping. We then propose a novel distance between curves to incorporate the object dynamics into the variational framework. For the minimization of the energy function, we developed a coordinate descent algorithm based on the level set methods. Experimental results for reconstructing a sequence of multiple dynamic objects are presented.

1. INTRODUCTION

The goal of dynamic tomography is to provide functional information of objects from the tomographic data observed over time. Its applications include cardiac computed tomography, dynamic studies in nuclear medicine such as myocardial perfusion imaging, blood pool imaging and renal imaging, etc. The challenges of dynamic tomography generally include: sparsity of projection data, high computational and memory requirement for this 3-D or 4-D problem, ill-posedness of the inverse problem, and the modeling of dynamics. Among these factors, the most challenging is the modeling of motion and its incorporation into the reconstruction process. Good temporal models will not only incorporate prior knowledge about scene dynamics, they will also reduce the dimension of unknowns and thus reduce the computational cost, and improve the robustness of the algorithm to noise.

In the field of nuclear medicine, considerable research has been done to model the dynamics of the image sequence at the pixel level. By using compartmental models from biological research, parametric models can be introduced to model the dynamic process at each pixel [1]. To reconstruct

the image sequence, a set of parameters then need to be estimated for every pixel. Along this line, various algorithms have been developed [2, 3]. However, these methods still results in high dimensional challenging optimization problems, and object-level motions are also hard to taken into account with these methods.

In many challenging tomography problems, the focus is on the extraction of information about a set of objects in the underlying scene. Direct estimation of parametric object models from projection data were first proposed in [4]. Recently, methods based on curve evolution and level set methods [5] become very popular for solving various static inverse problems [6, 7]. To this end, parametric object models have also been applied to reconstruct a single dynamic objects, for example [8, 9]. But these methods were not able to incorporate object motion models into the reconstruction.

In this paper, we propose a novel variational method for the reconstruction of dynamic objects directly from challenging tomographic data. We model the motion of objects explicitly and propose a novel shape distance to incorporate our object dynamic model into the reconstruction process. Our methods can reconstruct dynamic objects from extremely sparse projection data. Using efficient level set methods for object boundary representation, we can also reconstruct multiple dynamic objects simultaneously.

The rest of the paper is as follows. In section 2, we present our model of object dynamics. The variational framework is proposed in Section 3. Experimental results are presented in Section 4. Finally, conclusions are made in Section 5.

2. OBJECT-BASED MODELING

Let $f(\mathbf{x}, k)$ ($k = 1, 2, \dots, \mathbf{K}$) denote an image sequence consisting of $\mathbf{K}$ scenes and each scene is defined over a domain $\mathbf{D} \subset \mathbf{R}^2$. We assume the scene at time k is composed of N regions: $[\Omega_k^1, \dots, \Omega_k^N]$. The boundaries of these N regions can be represented by L ($L \leq N$) curves $[C_k^1, \dots, C_k^L]$. Since these curves can intersect, the number of regions they delineate is larger than the number of curves. The inten-

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sity of each region is assumed uniform and denoted as β_k^n . We stack the intensity of all regions into a vector $\beta_k = [\beta_k^1, \dots, \beta_k^N]$.

2.1. Dynamic model

Based on our image sequence model, we model the dynamics of the image sequence as composed of two parts: the dynamics of the object boundaries and the dynamics of the intensities.

For the dynamics of the object boundaries at time k , we model it as composed of two parts. A one to one and differentiable mapping $A_k : \mathbf{R}^2 \rightarrow \mathbf{R}^2$ is introduced to account for the part of the dynamics that is known a priori and another mapping $W_k : \mathbf{R}^2 \rightarrow \mathbf{R}^2$ is used to represent unknown and probably irreversible small perturbations to the boundary. Using these two transformations, we express the dynamics of the object boundary as follows:

$$C_{k+1}^l = A_k(C_k^l) + W_k, \quad l = 1, \dots, L \quad (1)$$

i.e., for any $\mathbf{x} \in C_k^l$, there is a point $\mathbf{x}' \in C_{k+1}^l$ such that $\mathbf{x}' = A_k(\mathbf{x}) + W_k(\mathbf{x})$.

For intensity dynamics, we use a simple auto-regressive model:

$$\beta_{k+1} = B_k \beta_k + u_k, \quad (2)$$

where B_k is the system matrix and u_k is a random vector of Gaussian distribution with zero mean and covariance matrix P_k .

2.2. Observation model

Let $Y_k^i (i = 1, \dots, M)$ denote the i -th component of the clean projection data at time k and $H_k^i(\cdot)$ the projection kernel. We can express the observation model as follows:

$$Y_k^i = \sum_{n=1}^N \beta_k^n \int_{\Omega_k^n} H_k^i(\mathbf{x}) d\mathbf{x} \quad (3)$$

We can also express this observation model in a vector form:

$$Y_k = H_k \beta_k \quad (4)$$

where H_k is an $M \times N$ matrix with its elements defined as: $H_k(i, n) = \int_{\Omega_k^n} H_k^i(\mathbf{x}) d\mathbf{x}$. Note that H_k is itself a function of $C_k^l (l = 1, \dots, L)$, so our dynamic tomography observation model is indeed *nonlinear* with respect to the unknowns.

In applications, noisy versions of the clean data are usually observed. Let y_k denote the noisy observation at time k . We assume a general noise model and express the conditional distribution of y_k given Y_k as:

$$y_k \sim \mathbf{p}(y_k | Y_k) \quad k = 1, 2, \dots, K \quad (5)$$

3. A VARIATIONAL FRAMEWORK

To reconstruct the image sequence directly from the sparse and noisy projection data, we propose a variational framework. With this approach, all the information and regularization terms can be combined into a single energy function. Tractable optimization algorithms can also be developed to estimate model parameters.

In our variational framework, an energy function E is defined as a weighted sum of four terms:

$$E = E_d + \lambda_s E_s + \lambda_t E_t + \lambda_i E_i, \quad (6)$$

where E_d is the data fidelity term, E_s is the spatial boundary smoothness term, E_t is the temporal boundary smoothness term, and E_i is the temporal intensity smoothness term. The details of their definition will be presented in the follows. λ_s , λ_t , and λ_i are non-negative regularization parameters used to weight these different terms. With the energy well defined, we formulate our reconstruction problem as an energy minimization problem:

$$(\mathbf{C}, \beta) = \arg \min_{(\mathbf{C}, \beta)} E(\mathbf{C}, \beta) \quad (7)$$

where $\mathbf{C} = [C_1^1, \dots, C_K^L]$ and $\beta = [\beta_1^1, \dots, \beta_K^N]$.

For the data fidelity term, we define E_d to be proportional to the negative log-likelihood of the data, i.e.:

$$E_d = -\log \mathbf{p}(y_k | Y_k) \quad (8)$$

Common choices of $\mathbf{p}$ are Gaussian and Poisson distributions.

The spatial boundary smoothness term E_s is defined as the sum of the length of all the boundaries:

$$E_s = \sum_{k,l} \int_{C_k^l} dC_k^l \quad (9)$$

By incorporating E_s as a regularization term, smoothness of the boundaries can be enforced [10].

The temporal boundary smoothness term E_t is designed to incorporate the boundary dynamics model in Eq.(1). This is a challenging task because W_k is unknown and correspondences between points on the two curves C_{k+1}^l and $A_k(C_k^l)$ are hard to establish in reality. Motivated by the L^p distance between vectors, we propose a novel distance between curves and penalize the distance between C_{k+1}^l and C_k^l to incorporate the boundary dynamics.

Let C_a and C_b be two closed curves in $\mathbf{R}^2$. Their difference region is $\pi(C_a, C_b)$ which are the set of points inside C_a but outside C_b or inside C_b but outside C_a . We define the distance of these two curves as:

$$d_p(C_a, C_b) = \left(\int_{\pi(C_a, C_b)} (d_{C_a}(\mathbf{x}) + d_{C_b}(\mathbf{x}))^{p-1} d\mathbf{x} \right)^{1/p} \quad (10)$$

where $p > 0$, $d_{C_a}(\mathbf{x})$ and $d_{C_b}(\mathbf{x})$ are the distance functions of C_a and C_b , respectively. Using our distance measure, the temporal boundary smoothness term E_t is:

$$E_t = \sum_{k,l} \left(\int_{\pi(C_{k+1}^l, \hat{C}_{k+1}^l)} (d_{C_{k+1}^l}(\mathbf{x}) + d_{\hat{C}_{k+1}^l}(\mathbf{x}))^{p-1} d\mathbf{x} \right)^{1/p} \quad (11)$$

where $\hat{C}_{k+1}^l = A_k(C_k^l)$, $d_{C_{k+1}^l}$ and $d_{\hat{C}_{k+1}^l}$ are the distance functions of the two curves C_{k+1}^l and $\hat{C}_{k+1}^l$.

The temporal intensity smoothness term E_i incorporates intensity dynamics defined in Eq.(2) and is chosen to be proportional to the negative log-likelihood of the joint distribution of the intensities, i.e:

$$E_i = -\log \mathbf{p}(\beta) \quad (12)$$

To minimize the energy function, we developed a coordinate descent algorithm:

- Choose a set of initial curves for $C_k^l (k = 1, \dots, K; l = 1, \dots, L)$;
- Assume the curves are known, minimize E with respect to β ;
- Assume the intensity is known, evolve all the curves in the gradient descent direction of E with numerical algorithms of level set methods [5];
- Return to Step 2 until convergence.

4. EXPERIMENTAL RESULTS

In this section, we apply our method to reconstruct an image sequence with multiple dynamic objects from sparse Poisson projection data.

The true image sequence was a simulated cardiac sequence generated from the NCAT phantom [11]. It consisted of 16 frames of size 128×128 covering a cardiac cycle. With multiphase level set methods [12], we used two level set functions at each time to represent four regions in the scene. Each region was assigned a binary label: the heart(01), the liver(10), the thorax region(00), and the background(11). Time varying intensities were also simulated for these four regions as shown in Fig.4. To generate the projection data, we used a SPECT projection matrix generated with the toolbox from [13]. Attenuation, depth dependent blurring, and Poisson noise were simulated. At time k , projection data were observed from 6 evenly spaced angles $[11k, 11k+30, 11k+60, 11k+90, 11k+120, 11k+150]$ degrees. The signal-to-noise ratio was 20dB.

The reconstructed image sequence is shown in Fig.2. The reconstructed boundaries are also plotted over the true image sequence in Fig.3 for comparisons. As we can see,

our method correctly reconstructed the boundaries of the left ventricle, the liver and the body contour. We also got very good estimation of the intensities as shown in Fig. 4.

Fig. 1. True image sequence at 4 time instants(k is the time index).

5. CONCLUSIONS

In this paper, we presented a variational framework for object-based dynamic tomography. A general object dynamic model was presented that can capture motion on the object level. This dynamic model was incorporated into the framework with a novel curve distance we proposed. Preliminary experimental results were presented.

6. REFERENCES

- [1] G.T. Gullberg, R.H. Huesman, S.G. Ross, E.V.R. Di Bella, G.L. Zeng, R.W. Reutter, P.E. Christian, and S.A. Foresti, "Dynamic cardiac single-photon emission computed tomography," in *Nuclear Cardiology: State of the Art and Future Directions*, B.L. Zaret and G.A. Beller, Eds., pp. 137–187. Mosby, 2nd edition, 1999.
- [2] R.E. Carson and K. Lange, "The EM parametric image reconstruction algorithm," *Journal of the American Statistical Association*, vol. 80, no. 389, pp. 20–22, MAR 1985.
- [3] T. Farncombe, A. Celler, C. Bever, D. Noll, J. Maeght, and R. Harrop, "The incorporation of organ uptake into dynamic SPECT(dSPECT) image recon-

Fig. 2. The reconstructed image sequence at 4 time instants(k is the time index).

Fig. 3. The reconstructed object boundaries compared with the true image sequence at 4 time instants(k is the time index).

struction,” *IEEE Trans. on Nuclear Science*, vol. 48, no. 1, pp. 3–9, FEB 2001.

- [4] D. J. Rossi and A. S. Willsky, “Reconstruction from projections based on detection and estimation of objects-parts I and II: Performance analysis and robustness analysis,” *IEEE Transactions on Acoustic, Speech, and Signal Processing*, vol. ASSP-32, no. 4, pp. 886–906, 1984.
- [5] S. Osher and J.A. Sethian, “Fronts propagation with curvature-dependent speed: algorithms based on Hamilton-Jacobi formulations,” *Journal of computational physics*, vol. 79, pp. 12–49, 1988.
- [6] A. Litman, D. Lesselier, and F. Santosa, “Reconstruction of a two-dimensional binary obstacle by controlled evolution of a level-set,” *Inverse Problems*, vol. 14, pp. 685–706, 1998.
- [7] H. Feng, W.C. Karl, and D.A. Castañón, “A curve evolution approach to object-based tomographic reconstruction,” *IEEE Trans. on Image Processing*, 2003.
- [8] P.C. Chiao, W.L. Rogers, N.H. Clinthorne, J.A. Fessler, and A.O. Hero, “Model-based estimation for dynamic cardiac studies using ECT,” *IEEE Trans. on Medical Imaging*, vol. 13, no. 2, pp. 217–226, JUN 1994.
- [9] G.S. Cunningham, K.M.Hanson, and X.L.Battle, “Three-dimensional reconstructions from low-count SPECT data using deformable models,” *Optics Express*, vol. 2, no. 6, pp. 227–236, 1998.

- [10] D. Mumford and J. Shah, “Boundary detection by minimizing functionals,” in *Proceedings of CVPR*, San Francisco, June 1985, pp. 22–26.
- [11] W.P. Segars, D.S. Lalush, and B.M.W. Tsui, “A realistic spline-based dynamic heart phantom,” *IEEE Trans. Nucl. Sci.*, vol. 46, pp. 503–506, 1999.
- [12] L.A. Vese and T.F. Chan, “A multiphase level set framework for image segmentation using the mumford and shah model,” *International Journal of Computer Vision*, vol. 50, no. 3, pp. 271–293, 2002.
- [13] J.A. Fessler, *ASPIRE 3.0 user’s guide: a sparse iterative reconstruction library*, 2002.

Fig. 4. The reconstructed intensities compared with the true intensities. Each region is identified with a binary label.