Distributed Target tracking and localization in multi-hop networks

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***Abstract*— We consider the problem of distributed target tracking under communication constraints between the sensor nodes, a problem that has recently received significant attention. On account of communication constraints the problem necessitates not only the dynamic selection of optimal sensor nodes but also corresponding fusion centers to enable local processing of sensor information. This coupled problem is generally intractable and significant effort has been devoted towards proposing greedy strategies under various performance criteria. In contrast our paper decouples the three aspects, namely non-linear partially observed state estimation, sensor selection and fusion center locationing, and adopts a certainty equivalence perspective. The main advantage of this approach is that since significant communication costs in target tracking with multi-hop networks arises primarily from the switching of fusion centers, this problem can be isolated and optimized. In particular, we show that optimal tracking algorithms that jointly optimizes the average multi-hop communications as well as average tracking error can be derived in an infinite horizon setting. Specifically, when target dynamics is described by a random walk, the optimal strategy exhibits a hybrid switching strategy, whereby active sensor locations are held stationary until the target moves outside a threshold radius around the sensors. This holds even when there are sensors located close to the target. Surprisingly this result is fundamental to the multi-hop communication penalty and does not hold for penalties that reflect free space attenuation.**

I. INTRODUCTION

We consider the problem of tracking a target moving in region populated by sensor nodes that have limited wireless communication capabilities. The sensors have limited sensing range and the quality of range estimates degrades with distance from the target. The best accuracy is obviously obtained if the measurements from all the sensors can be processed jointly to estimate target location. Nevertheless, this centralization is not possible due to communication constraints on individual sensors. To accomplish this task a distributed mechanism for target tracking is necessary.

Our distributed mechanism can be described as follows: We assume that there is no central authority governing network operation. Since information degrades with distance, only sensors sufficiently close to the target share their data. Second, since there is no designated central authority an arbitrary sensor is designated as the fusion center (or *leader node*) for a given time period and the data is processed at this sensor. Therefore, the entire mechanism can be broken down into three tasks:

1) Given the current sensor set, fusion center and past target information, the fusion center estimates the current target position based on sensed information from sensor set.
2) Fusion center then computes the new sensor set as well as the new fusion center for the next time period based on current estimated target state. Based on communication costs the fusion center may decide not to switch the current set of sensors.
3) Fusion center then communicates state information to the new fusion center.

To summarize communication costs not only arise while transmitting data from sensors set to the fusion center but also while transmitting current state information to a subsequent fusion center. Therefore, the fusion center must not only be located sufficiently close to the informative sensors but the distance from a fusion center to the next fusion center must be sufficiently small. For accurate tracking the sensors have to be selected sufficiently close to the target. This implies that for rapidly evolving trajectories the active sensor locations will need to move rapidly as well. But this in turn increases communication costs and problem is to find strategies that balance communication costs with accuracy.

For most tracking situations a Markov model is assumed for target dynamics with noisy state observation (non-linear). The process noise and the observation noise are independent of the current state and current observation. Under the Markovian model, Task 1 can be performed in a recursive way using only the past estimate and the current set of observations, [1]. Task 2 in the above set-up is a policy that is critical to the decentralized tracking performance. It is to be noted that the observation matrix (or function) in the next time period, depends on the policy for sensor selection since the measurements are a function of the actual target location and the location of the sensors. Consequently, the error in the state estimate is *not independent* of the past control actions taken. Under these circumstances it is well known, [1], that separation of estimation and control in the conventional sense is not optimal. The problem requires setting up of the so called 'information state' and leads to explosion of the dimensionality of the problem. Task 3 in the above set-up is subject to communication constraints, i.e. there is a cost to hand-off the current estimate to the next set of sensors and there is a cost of aggregating the measurements at the fusion

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center (or *leader node*). This in turn also determines the policies for Task 2 which affects the estimation performance. The problem in general is intractable for even reasonable number of time steps.

To better focus the problem we formulate a tractable formulation that retains the essential features of the original problem, which is to understand the inherent tradeoff's between the communication costs and the estimation performance in such distributed tracking problems. We concentrate primarily on Task 2 namely selection of sensors for a related fully observed markov decision problem. The main advantage of this approach is that since significant communication costs in multi-hop networks arises primarily from the switching of fusion centers and sensor set, this problem can be isolated and optimized. In particular, we show that optimal tracking algorithms that jointly optimizes the average multi-hop communications as well as average tracking error can be derived in an infinite horizon setting. Specifically, we show that if the target dynamics is described by a random walk with sufficiently small mean drift, the optimal strategy exhibits a hybrid switching strategy characterized by deadzones. The sensor locations are stationary until the target exits the deadzone and then the locations are switched to best sensor locations around the target. In order to compare these results with existing approaches we adopt a certainty equivalent perspective to deal with partially observed case as well as the discrete sensor geometry. Our results show significant improvements over current approaches in energy savings.

II. General Problem

The set of sensors is denoted by $\mathcal{S} = \{s_1, s_2, ..., s_n\}$. At any time t the observation at the sensor s_i is given by,

$$Z_{s_i}(t) = \frac{A}{1 + ||x(t) - x_{s_i}||_2} + V_{s_i}$$

where A is the amplitude of the signal originating from the target and x_{s_i} is the position of sensor s_i. V_{s_i} is an additive Gaussian noise of variance σ_v and is independent of $x(t)$. The target dynamics are given by the following dynamical model

$$x(t+1) = f_t(x(t+1)) + w(t)$$

where $w(t)$ is an independent zero mean Gaussian noise of variance σ_w.

Under the proposed distributed tracking framework, let $l(t)$ denote the set of active sensors at time t. Let $\hat{x}(t)$ denote the current state estimate based on the observations collected till time t. Let $u(t) = u(\hat{x}(t), l(t)) = l(t+1) \subset \mathcal{S}$, be the control action at time t that selects the next set of active sensors. Let the cost at any time t be denoted by $g(t) = g(\hat{x}(t), l(t), u(t))$. Let C_0 denote the total communication budget. With slight abuse of notation we use $l(t)$ to also denote the leader node. Then we have the following N horizon optimization problem, (P).

(P) :

$$\min_{u(1),..,u(N-1)} \ \sum_{t=1}^{N-1} \ g(\hat{x}(t), l(t), u(t)) + g(\hat{x}(N), l(N))$$

$$\text{sub to} : \sum_{t=1}^{N-1} c_1(u(t) - l(t)) + c_2(l(t)) + c_2(l(N)) \leq C_0$$

where c_1 is the cost of *hand-off* and c_2 is the cost of communication in aggregating the measurements at the leader node.

In the absence of communication constraints, the problem reduces to selecting the best sensors for each period of time, as described by Task 2. Several papers, [2], [3], [4], have attempted to address this problem using information theoretic costs on the estimation performance. There the communication was restricted to be multi-hop. Heuristic and greedy (one-step optimal) strategies were proposed to reduce problem dimensionality. The main difficulty with sensor selection approaches is their combinatorial complexity.

In presence of communication constraints the N horizon optimization problem (P) subject to total communication constraints has been formulated in [5].

In [5] the cost $g(\hat{x}(t), l(t), u(t))$ was taken to be negative of the conditional mutual information between the updated state $\hat{x}(t)$ and the observations from the nodes in $l(t+1)$ conditioned on all the past information, i.e.,

$$g(\hat{x}(t), l(t), u(t)) = -I(\hat{x}(t); Z(l(t+1))|Z_0^t)$$

where $Z(l(t))$ denote the set of observations from sensors in $l(t)$. The communication costs c_1 and c_2 are $c_1(u(t) - l(t)) = ||u(t) - l(t)||^2$, and c_2 is the cost of communication from the nodes in the subset to the leader node. Note that c_2 is essentially a constant cost on top of the cost of *hand-off*.

Inherently, as stated before, the sensor selection criteria in the above set-up has combinatorial complexity. Moreover, due to the inherent nature of hidden observations this optimization problem suffers from the usual dimensional explosion of the state, which is commonly referred to as the information state. To deal with these issues a greedy algorithm was proposed in [5]. Nevertheless, a tradeoff between the communication costs and the estimation error is still not apparent.

III. Simpler Problem

To get insight into the fundamental tradeoffs between communication and estimation performance we formulate a simpler problem, which nevertheless has the essential features of the original problem. We adopt the following strategy with the accompanying assumptions:

1) We adopt a certainty equivalence perspective and pose an analytically tractable problem for target tracking. In particular, we de-couple state estimation and control, which reduces the complexity of Task 2.

2) We assume a uniform placement of sensor nodes in the sensing region. This implies the set of most informative sensors is typically a small subset of sensors around a small radius around the target position.
3) The quality of observations degrades substantially with increasing sensor distance from the target.
4) The communications cost can be approximated by distance of the fusion center to the target. This follows because of two equivalent possibilities: (a) fusion center is not updated but the sensor set is updated to get information close to the target, (b) both fusion center and sensor set are close to target. In the former case although there is no communication penalty for communicating past information, there is a high cost to communicating sensor information. Since the number of active sensors at any time is a constant this cost is a constant multiple of scenario (b).
5) Since hidden observations is an issue due to inherent problems with partially observed markov decision processes even in the centralized situation we limit ourselves to fully observed scenario.

a) Multi-hop Communication Costs & ℓ_1 penalty: Although the communication is directly proportional to the square (or fourth power) of the distance of communication, this penalty is not truly reflective of meshed networks where multi-hop costs dominate. In a low power sensor network where the *communication protocol* is restricted to be multi-hop, the communication cost is dominated by the number of hops taken from the source to destination, in this case from current leader node to the next one. This means that it depends mainly on the absolute distance (that determines the number of hops) and not on the squared distance between the nodes. Thus ℓ_1 norm on the communication distance appears to account for communication in a multi-hop network. Restriction to multi-hop operation is due to the fact that the energy requirements for multi-hop operation is proportional to number of hops taken, whereas in for direct communication the energy requirements are proportional to d^α where α is the attenuation factor,($\alpha = 2$) for free space). This essentially amounts to significantly larger communication energy.

To simplify our analysis we assume a continuum of sensors, i.e., there are sensors everywhere at any point in the region. We identify each sensor with its location. With slight abuse of notation a leader node will be denoted by $l(t)$ which also denotes the location of the sensor. The error in state estimation is inversely proportional to the power received, and is thus proportional to $||x(t) - l(t)||^2$ for a free space wave propagation model. The decision variable at time t is the location of the next active sensor, denoted by $l(t+1)$, based on the current position $x(t)$ of the target. Let $u(t) = l(t+1) - l(t)$. Then the cost of communication is proportional to $||u(t)||_1$ for multi-hop networks and proportional to $||u(t)||_2^2$ otherwise. Assuming the proportionality constants to be unity we have the following two candidate unconstrained optimization problems.

$$Q1 : \min_{u(1),..,u(N-1)} \sum_{t=1}^{N-1} \left[\mathbf{E}||u(t)||_1 + \mathbf{E}||x(t) - l(t)||^2\right] + ||x(N) - l(N)||^2$$

$$Q2 : \min_{u(1),..,u(N-1)} \sum_{t=1}^{N-1} \left[\mathbf{E}||u(t)||^2 + \mathbf{E}||x(t) - l(t)||^2\right] + ||x(N) - l(N)||^2$$

where the expectation is over the process noise in the target dynamics.

These two simple models assume that there is continuum of sensors and that the state of the target is noiselessly observable.

A. One-dimensional target dynamics

In the following we will present analysis for one dimensional target dynamics. The results extend to the two dimensional case in a straightforward way as the minimization at each step is decoupled in different dimensions.
For this we will assume a simple target dynamics model. Let the target dynamics be given by

$$x(t+1) = x(t) + w(t)$$

where $w(t)$ is a zero mean noise independent of $x(t)$ and $x(0) = 0$. If $w(t)$ is zero mean Gaussian random variable, then $x(t)$ is the standard Brownian motion starting at zero. Let the state of the system (sensor network) be given by the $\tilde{x}(t) = [x(t), l(t)]'$ i.e. the current state of the target and the current activated node. Then we have the following dynamics for the extended state space.

$$\tilde{x}(t+1) = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \tilde{x}(t) + \begin{bmatrix} 0 \\ I \end{bmatrix} u(t) + \begin{bmatrix} I \\ 0 \end{bmatrix} w(t) \quad (1)$$

Effectively the state of the system can also be captured by $z = x(t) - l(t)$. Then the system equation evolves according to the function

$$z(t+1) = z(t) - u(t) + w(t)$$

Under the above simple dynamics we have the following optimization problems to solve.

$$Q1 : \min_{u(1),..,u(N-1)} E\left(\sum_{t=1}^{N-1} \left[||u(t)||_1 + ||z(t)||^2\right] + ||z(N)||^2\right)$$

$$Q2 : \min_{u(1),..,u(N-1)} E\left(\sum_{t=1}^{N-1} \left[||u(t)||^2 + ||z(t)||^2\right] + ||z(N)||^2\right)$$

IV. Policy under ℓ_2 Communication Model

We consider first the case of quadratic penalty on the control action, i.e., communication cost. Under the state dynamics model for $z(t)$ the optimization problem $Q2$ is the standard LQ-controller, under perfect state observation, the solution to which is known, [1]. At any time t the optimal control action is of the form $u(t) = -K(t)z(t)$. Notice that if $z(t)$ is a Brownian motion in the absence of control, then the optimal control is a scaled Brownian motion. This also implies that one has to *switch* to a new leader node all the time. Under multi-hop operation the communication cost—which amounts to the total variation of the scaled Brownian motion—goes to infinity. Thus for multi-hop networks an ℓ_2 constraint does not reflect the true communication costs.

V. Policy under ℓ_1 Communication penalty (Multi-hop)

To account for multi-hop communication costs we penalize communication costs through an ℓ_1 cost criterion. For sake of exposition we consider two cases, viz., (a) no process noise, i.e., a stationary target and (b) a uniform process noise.

A. No Noise Case

For no process noise and perfect state observation, consider the cost to go at time $N-1$,

$$J_{N-1}(x_{N-1}) = \min_{u_{N-1}} \left\{ |u_{N-1}| + z_{N-1}^2 + (z_{N-1} - u_{N-1})^2 \right\}$$

The derivative of $|u_{N-1}|$ is not defined at zero. For this we will find the subgradient set of the function inside the minimization. At zero the subgradient set of $|u_{N-1}|$ is given by $[-1, 1]$. Thus if $|z_{N-1}| < 1/2$ then zero is an element of the subgradient set. This implies that optimal control $u_{N-1} = 0$ if $|z_{N-1}| < 1/2$. Then the optimal cost to go at stage $N-1$ is given by

$$J_{N-1}(z_{N-1}) = \begin{cases} 2z_{N-1}^2 & \text{if } |z_{N-1}| \leq 1/2 \\ |z_{N-1} - 1/4| + z_{N-1}^2 & \text{if } |z_{N-1}| > 1/2 \end{cases}$$

The interpretation of the control at N-1 is that we do not apply any control if the state z_{N-1} is in a "dead-zone" region $[-1/2, 1/2]$.

It can be shown that at time $N-k$ the cost to go function is given by

$$J_{N-k}(z_{N-k}) = \begin{cases} (k+1)z_{N-k}^2 & |z_{N-k}| \leq 1/2k \\ |z_{N-k} - \frac{1}{2k}| + z_{N-k}^2 + \frac{k}{4k^2} & |z_{N-k}| > 1/2k \end{cases}$$

and the optimal "dead-zone" region at stage $N-k$ is given by $[-1/2k, 1/2k]$. Note that the cost to go is a smooth convex function. The dead-zone region shrinks. This is reflective of the fact that at any stage if the state $z(t) = x(t) - l(t)$ is below a certain threshold then no control is applied. This result can be used to effectively localize a stationary target with minimal communication costs in a multi-hop network.

B. Uniform bounded process noise

Again for the sake of analytical tractability we assume that the noise process $w(t)$ is a uniform noise bounded between $[-\alpha, \alpha]$. In this case we have,

$$J_{N-1}(z_{N-1}) = \min_{u_{N-1}} \left\{ |u_{N-1}| + z_{N-1}^2 + E_w(z_{N-1} - u_{N-1} + w_{N-1})^2 \right\}$$

Since the noise is zero mean and uncorrelated, the cost to go at $N-1$ is given by,

$$J_{N-1}(z_{N-1}) = \begin{cases} 2z_{N-1}^2 + \sigma_w^2 & |z_{N-1}| \leq 1/2 \\ |z_{N-1} - 1/4| + z_{N-1}^2 + \sigma_w^2 & |z_{N-1}| > 1/2 \end{cases}$$

We now have the following surprising result.

Theorem 5.1: The cost to go J_{N-k} is convex, symmetric and contains a non-vanishing dead-zone for any stage $N-k$, i.e. the optimal control at stage $N-k$ is zero for $|z_{N-k}| \leq \Gamma_{N-k}$ where $\Gamma_{N-k} \geq \Gamma_* > 0, \ \forall k \in Z^+$.

Proof: See Appendix. ■

For purpose of illustration, we choose a uniform noise between $[-0.25, 0.25]$. The optimal cost to go for various stages and the optimal control at various stages are shown in figure 1. This implies that the optimal stationary policy is described by a footprint region corresponding to each sensor node. The processing is localized at a node as long as the target remains within that nodes footprint and the processing center is switched as soon as the target leaves this region. Observe that this strategy is specific to multi-hop networks due to the ℓ_1 penalty on communication costs.

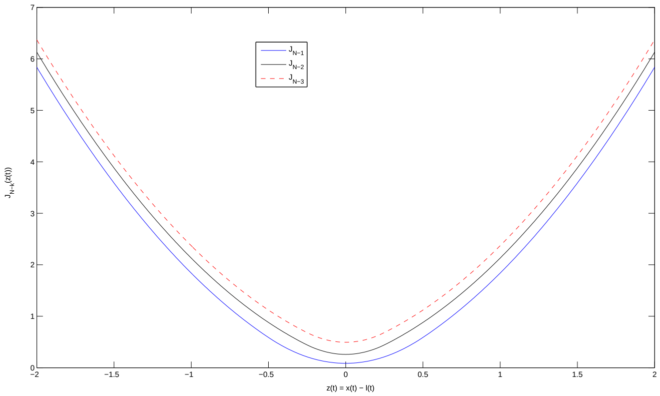
VI. Simulations

The simulation set up is as follows. The sensor network is a sensor grid consisting of 81 sensors placed on integer co-ordinates in $[-3, 3] \times [-3, 3]$. For the Brownian motion, uniform noise between [-0.25,25] was chosen as the process noise. The initial state was at [0,0] with initial covariance of 10 in each direction. The observation model at each sensor is given by,

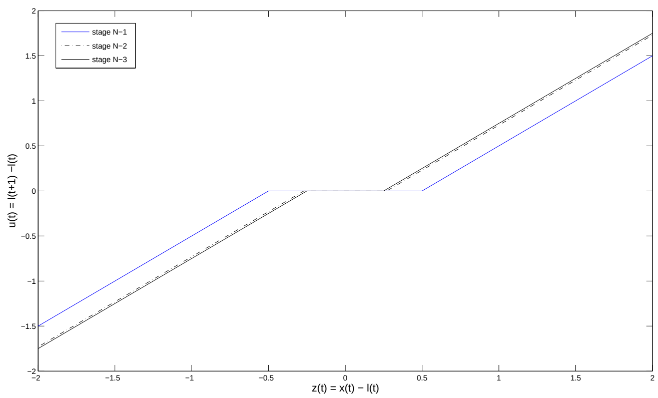
$$y(x_p, y_p) = \frac{20}{1 + \sqrt{2}(((X - x_p)^2 + (Y - y_p)^2))} + v$$

where v is a gaussian noise of variance 1 in each dimension and x_p and y_p are the x and y co-ordinates of the sensor. The switching threshold was kept constant to 0.2 over the 50 time steps. We run two algorithms.

1. At each time instant 4 nodes around the leader nodes report their observations. We use an Extended Kalman Filter (EKF) to update/predict the state. We employ certainty equivalence control where the control action for switching uses the state estimate given by the EKF. The leader node is switched to the predicted location if the distance between the current leader node and the current predicted position given by the EKF exceeds 0.2.
2. We compare the performance of the above algorithm to a hindsight policy where each time 4 sensors around the true target position were selected to report to the leader



(a)



(b)

Fig. 1. (a) The optimal cost to go for three stages in the backward DP recursion. (b) The optimal control policy as a function of the state $z(t)$ for various stages of DP recursion. Notice that the dead zone region in this case does not shrink to zero.

node. The leader node then uses an EKF for estimating the target position. This policy reflects the optimal state estimate with 4 sensors since the optimal sensor locations are provided as side information. However, it does not account for the communication cost.

The communication cost (in bit-meters) for switching is given by the total distance traveled by the leader node in 50 time steps. The results are shown in figure 3 where the statistics are averaged over 1000 Monte Carlo runs. As seen the tracking error for both schemes is similar. However the communication cost for hindsight EKF policy turns out to be 12.3 bit-meters while communication cost for our deadzone adaptive switch is 4.2bit-meters, thus providing a significant energy savings even when compared with hindsight EKF. Shown in figure 2 is the error performance Vs the communication cost. The communication costs are varied via varying the threshold. Figure 3 shows the rms error plots for the hindsight and the adaptive policy and shows x and y position tracking.

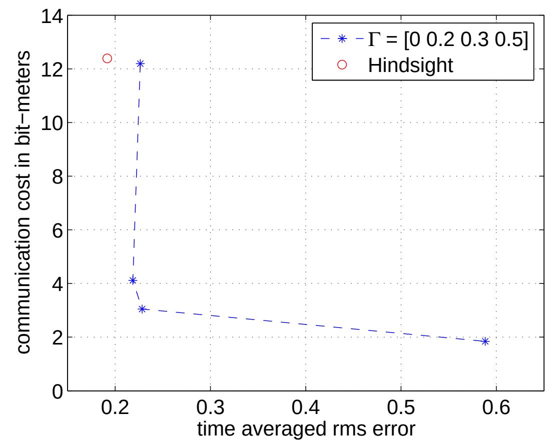
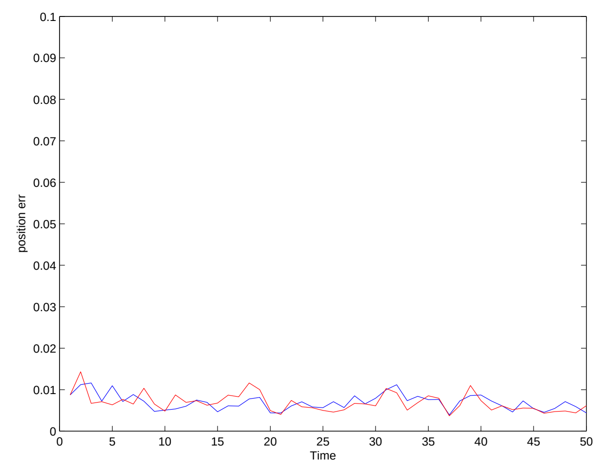
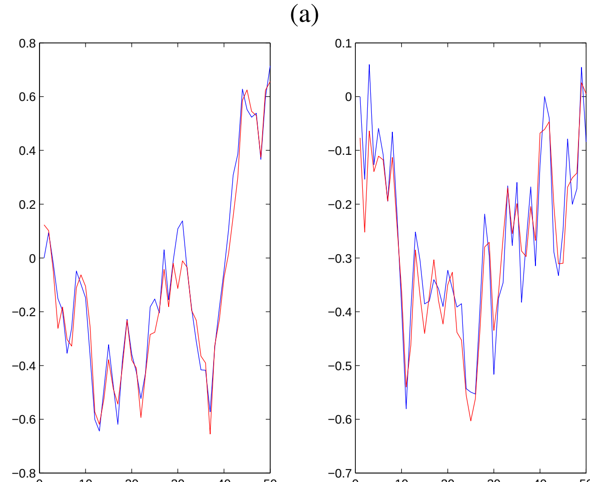


Fig. 2. The tradeoff between the communication costs and the error performance for the Brownian motion case. Note that the error performance does not degrade by much for $1/3$ of the switching costs. As Γ is increased the communication costs go down, but the error increases.



(a)



(b)

Fig. 3. (a) The rms errors for the adaptive switching center EKF and the optimized hindsight EKF. (b) The tracking of x and y positions with adaptive EKF for one of the target tracks

VII. Appendix

A. Proof of theorem 5.1

We establish the proof through induction. Let n denote the number of stages left. Then we have

$$J_n(z_n) = \min_{u_n} \left\{ |u_n| + z_n^2 + \mathbf{E}(J_{n-1}(z_n + w_n - u_n)) \right\}$$

The cost to go J_n is convex, symmetric and is monotone increasing in $z_1 \in \mathbb{R}^+$ and the derivative is zero at $z_1 = 0$ for $n = 1$. Suppose these properties also hold also for stage $n-1$. Then since the joint function on the, right hand side in the above recursion is jointly convex in z_n and u_n, it follows that $J_n(z_n)$ is a convex function in z_n (This follows from a theorem in Rockefeller [6] the fact that minimizing a jointly convex function in two variables results in a convex function in one variable, i.e., $f^*(x) = \min_y f(x, y)$ is a convex function of x). The symmetry is obvious.

Next we establish that the structure of the optimal policy at any stage is given by a sequence of non vanishing numbers $\Gamma_n > 0$ such that $u_n^* = 0$ if $|z_n| \leq \Gamma_n$. For this let us find the conditions for $u_n^* = 0$. The sub gradient set (under u_n) of the function under minimization at $u_n = 0$ is given by,

$$[-1, 1] + \left\{ \frac{\partial}{\partial u_n} \mathbf{E}(J_{n-1}(z_n + w_n - u_n)) \right\}_{u_n=0}$$

Consider,

$$\left\{ \frac{\partial}{\partial u_n} \mathbf{E}(J_{n-1}(z_n + w_n - u_n)) \right\}_{u_n=0}$$

$$= \lim_{\epsilon_n \downarrow 0} \frac{\mathbf{E}J_{n-1}(z_n + w_n - \epsilon_n) - \mathbf{E}J_{n-1}(z_n + w_n)}{\epsilon_n}$$

$$= \frac{1}{2\alpha} \lim_{\epsilon_n \downarrow 0} \int_{-\alpha}^{\alpha} \frac{J_{n-1}(z_n + w_n - \epsilon_n) - J_{n-1}(z_n + w_n)}{\epsilon_n} dw_n$$

$$= \frac{1}{2\alpha} \lim_{\epsilon_n \downarrow 0} \int_{\alpha}^{\alpha} f_n dw_n$$

Note that since the function J_{n-1} is convex in its argument the sequence of functions f_n is monotonic and has a limit (the derivative of J_{n-1} is defined everywhere). Thus by monotone convergence theorem one can exchange the limit and the integration. Thus we have for the sub gradient set at $u_n = 0$,

$$= [-1, 1] - \frac{1}{2\alpha} \int_{-\alpha}^{\alpha} \frac{\partial}{\partial z_n} J_{n-1}(z_n + w_n) dw_n$$

Now note that by induction hypothesis J_{n-1} is a symmetric convex function with derivative at $z = 0$ equal zero. This implies that the derivative of J_{n-1} is an antisymmetric function. The integral above is a convolution operation. Note that since the function is convex the derivative is an increasing function. Now note that convolution of a symmetric function with an antisymmetric function is antisymmetric. This implies that there exists a Γ_n such that for all $|z_n| \leq \Gamma_n$, $0 \in$ subgradient set. Given that such a region exists we have for the cost to go

$$J_n(z_n) = z_n^2 + \mathbf{E}(J_{n-1}(z_n + w)), \ \forall |z_n| \leq \Gamma_n$$

Note that it is symmetric in the region and the derivative is zero at $z_n = 0$. Thus this property holds for all n by induction.

At $u_n \neq 0$, the sub gradient is given by (for $u_n > 0$),

$$1 - \mathbf{E}(\frac{\partial}{\partial x} J_{n-1}(x + w))$$

Again $\mathbf{E}(\frac{\partial}{\partial x} J_{n-1}(x + w))$ is antisymmetric in x. For some x^*,$1 - \mathbf{E}(\frac{\partial}{\partial x} J_{n-1}(x^* + w)) = 0$. This implies that for some $z_n - u_n^* = \Gamma_n, \forall |z_n| > \Gamma_n$. Thus we have,

$$J_n(z_n) = |z_n| - \Gamma_n + z_n^2 + \mathbf{E}J_{n-1}(\Gamma_n + w), \ \forall |z_n| > \Gamma_n$$

Thus we have,

$$J_n(z_n) = \begin{cases} z_n^2 + \mathbf{E}(J_{n-1}(z_n + w)) & |z_n| \leq \Gamma_n \\ |z_n| - \Gamma_n + z_n^2 + \mathbf{E}J_{n-1}(\Gamma_n + w) & |z_n| > \Gamma_n \end{cases}$$

It is clear that $J_n(z_n)$ is symmetric, convex function with derivative at zero equal to zero. Thus by induction hypothesis it holds for all n. Now we are left to show that $\Gamma_n > 0$ for all n. For this it is sufficient to show that for all n the derivative $\frac{\partial}{\partial z_n} J_n(z_n)$ is bounded near zero. ¿From the recursive relation for the cost to function, we have that

$$\frac{\partial}{\partial z_n} J_n(z_n) = \begin{cases} 2z_n + 1 & |z_n| > \Gamma_n \\ 2z_n + \mathbf{E}\frac{\partial}{\partial z_n} J_{n-1}(z_n + w_n) & |z_n| \leq \Gamma_n \end{cases}$$

In an ϵ neighborhood of $z_n = 0$, for sufficiently large α, the value of the slope is given by

$$\begin{aligned} \frac{\partial}{\partial z_n} J_n(z_n)|_{[-\epsilon, \epsilon]} &\leq 2\epsilon + \frac{2}{2\alpha} \int_{\alpha}^{\alpha+\epsilon} 2x + 1 dx \\ &= (4 + \frac{1}{\alpha})\epsilon + \frac{\epsilon^2}{\alpha} \end{aligned}$$

if $\frac{\epsilon}{\alpha} \leq 1$ then the derivative is bounded by

$$\frac{\partial}{\partial z_n} J_n(z_n)|_{[-\epsilon, \epsilon]} \leq (5 + \frac{1}{\alpha})\epsilon$$

Thus the proof is complete.

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