On the Number of Bits to Encode the Outputs of Densely Deployed Sensors

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Collaborators

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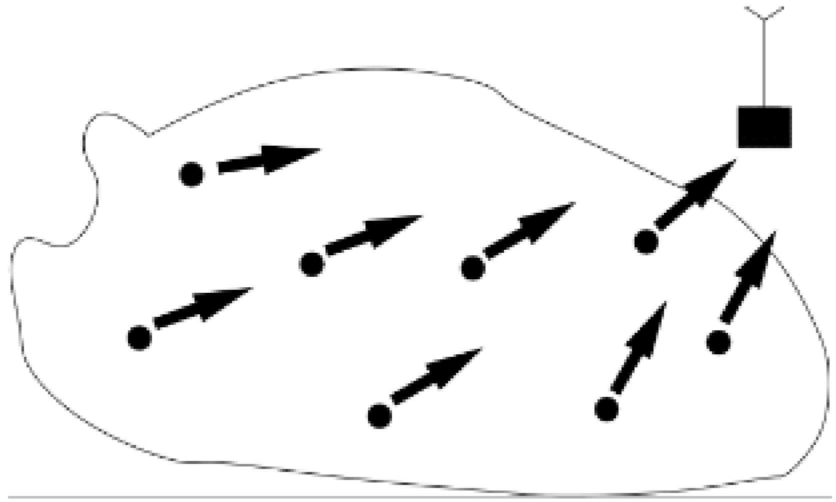
Sensors Sample a Stationary Random Field

Scenario:

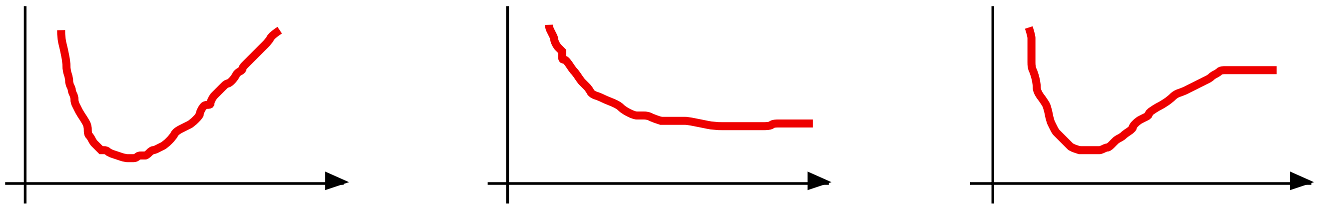


Sample a stationary random field with N sensors per unit area. The outputs are encoded, with no intersensor cooperation, in such a way that a reconstruction of the field with distortion d is possible.

Question:

As N increases, what happens to total # bits produced by encoders?

$$R_N(d) \;=\; N \times \#\,\text{bits/sensor sample} \;\to\; ????$$



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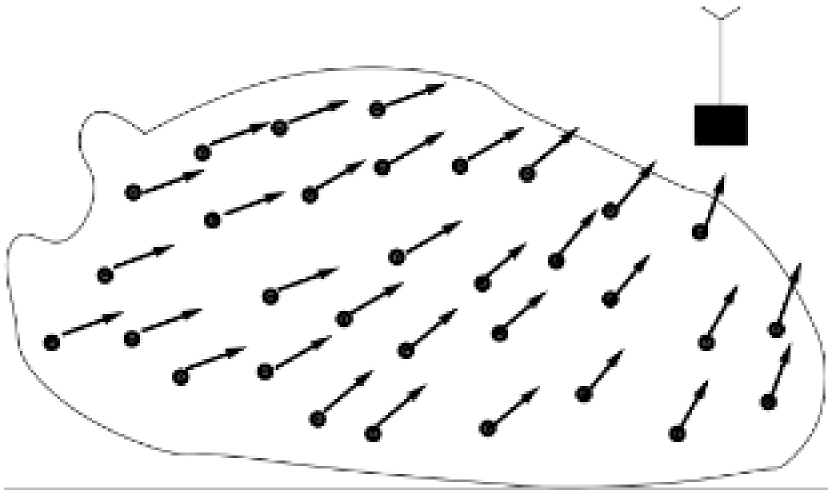

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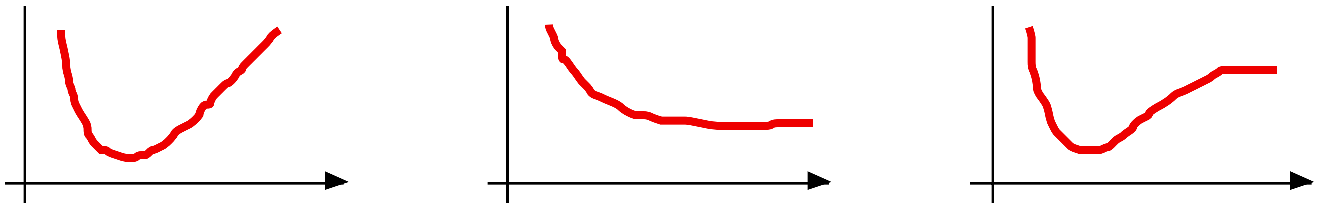


SEVERAL PARTIAL ANSWERS

- Mostly for bounded, deterministic, bandlimited signals, distortion $\rightarrow 0$ as $N \rightarrow \infty$

  a. Cvetkovic, Vetterli, *IEEE IT 1998, 2001*

  b. Cvetkovic, Daubechies, *DCC 2000*

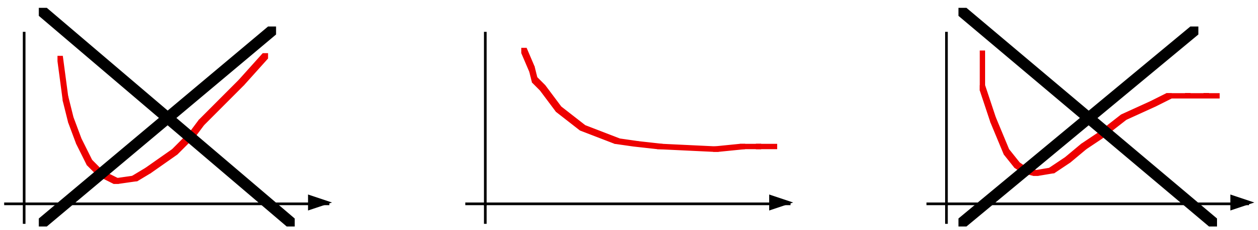
  c. Ishwar, Kumar, Ramchandran, *IPSN 2003, 2004*

- For unbounded, random, bandlimited or not signals distortion fixed as $N \rightarrow \infty$

  1. Marco, Duarte-Melo, Liu, Neuhoff, *IPSN 2003*
     See also, Marco, Neuhoff, *ISIT 2005*
  2. Kashyap, Lastras-Montana, Xia, Liu, *DCC 2005*
  3. Neuhoff, Pradhan, *ICASSP 2006*

IF WE ALLOWED ENCODER COOPERATION

- The minimum rate would be given by Shannon's rate-distortion function for continuous-space random field, which is finite.

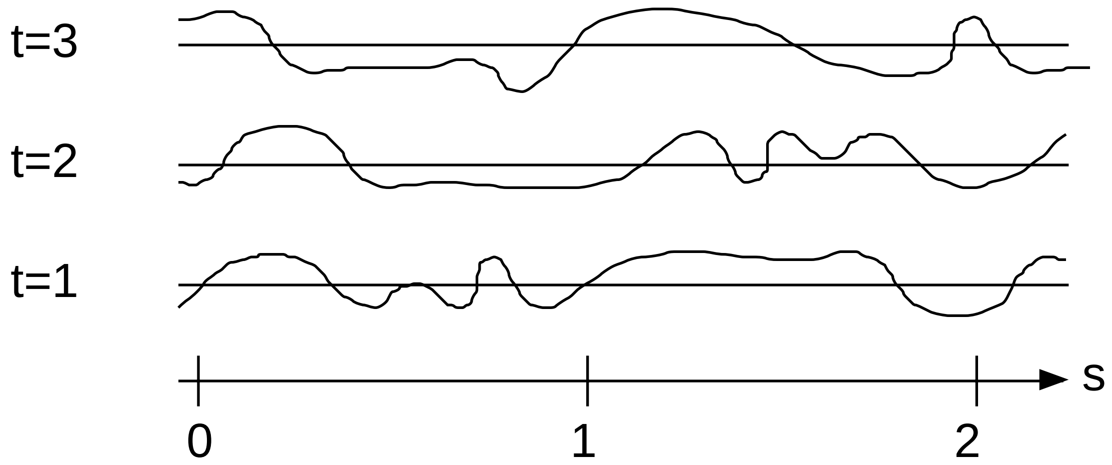


- But in a sensor network we need a distributed/decentralized encoder.

One-Dimensional Field Model

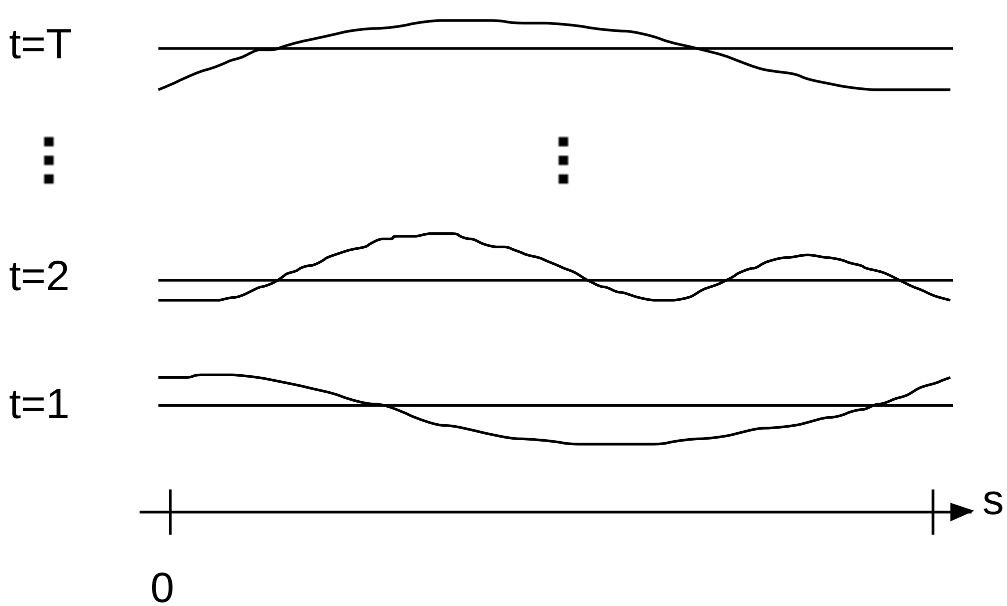
- Source: 1-dim'l stat'ry cont-space random process X(s), $-\infty < s < \infty$
- One independent sample function of X(s) every second.

  That is, for t = 1,2,...,   X(t,s) is an independent realization of X(s).



SAMPLE AND CODE

- Integer sampling rate: N samples/meter, i.e. sample every $\frac{1}{N}$, $\quad Y_n^{(t)} = X(t,\frac{n}{N})$
- **Encode** $Y_n^{(t)}$ into R_N bits/sample, $\quad \mathsf{R}_N = N\, R_N$ bits/meter
- **Decode** bits into $\hat{Y}_n^{(t)}$
- Linear interpolation: $\hat{X}(t,s) = \sum_{n=-\infty}^{\infty} \hat{Y}_n^{(t)}\, \pi_N(s-\frac{n}{N})$



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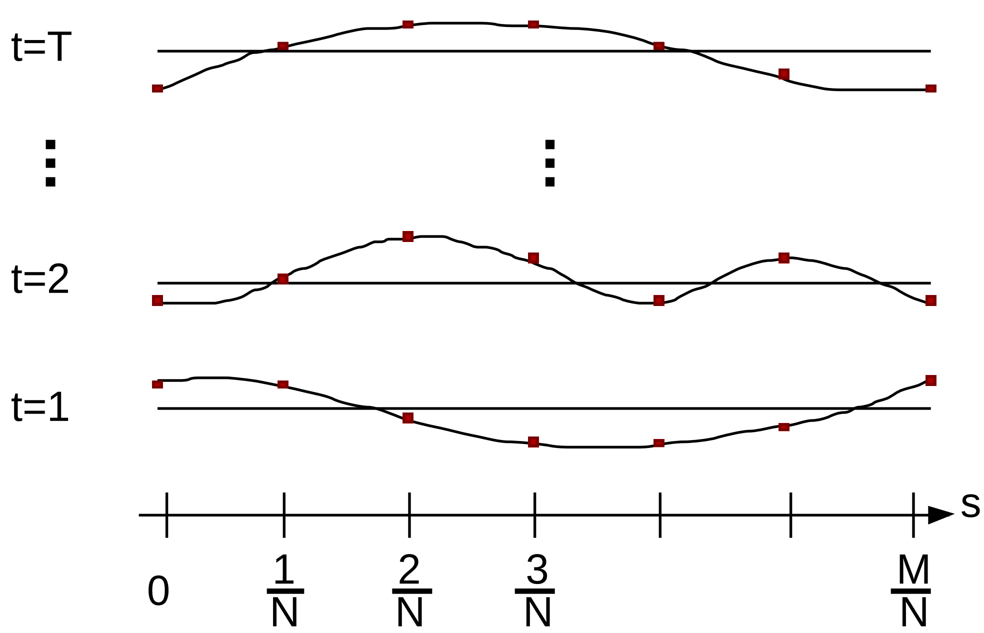

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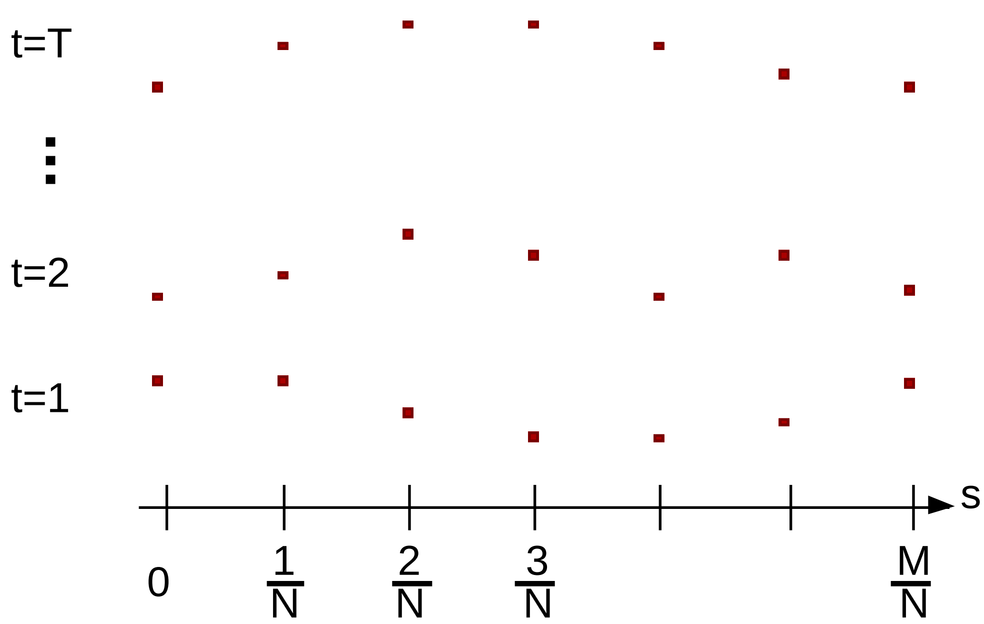

SAMPLE AND CODE

- Integer sampling rate: N samples/meter, i.e. sample every $\frac{1}{N}$, $Y_n^{(t)} = X(t,\frac{n}{N})$
- Distributed **Encode** $Y_n^{(t)}$ into R_N bits/sample, $R_N = N\, R_N$ bits/meter
- Jointly **Decode** bits into $\hat{Y}_n^{(t)}$
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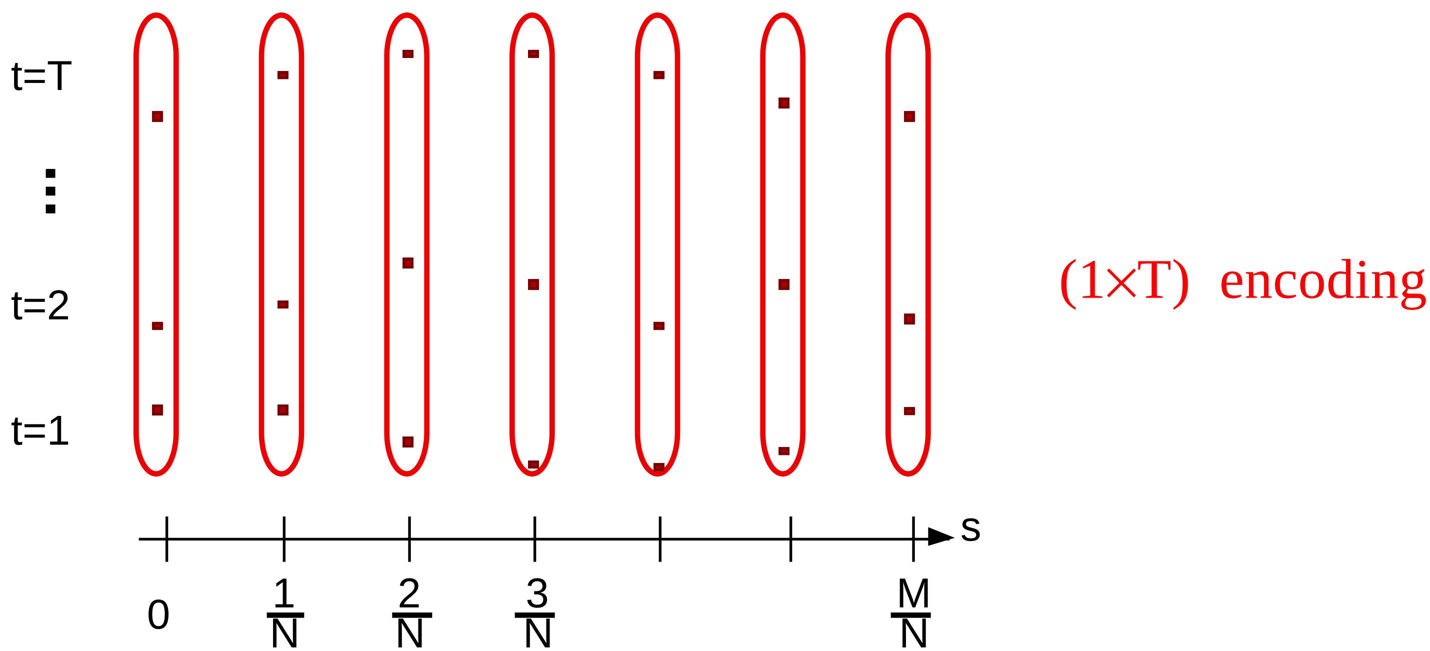

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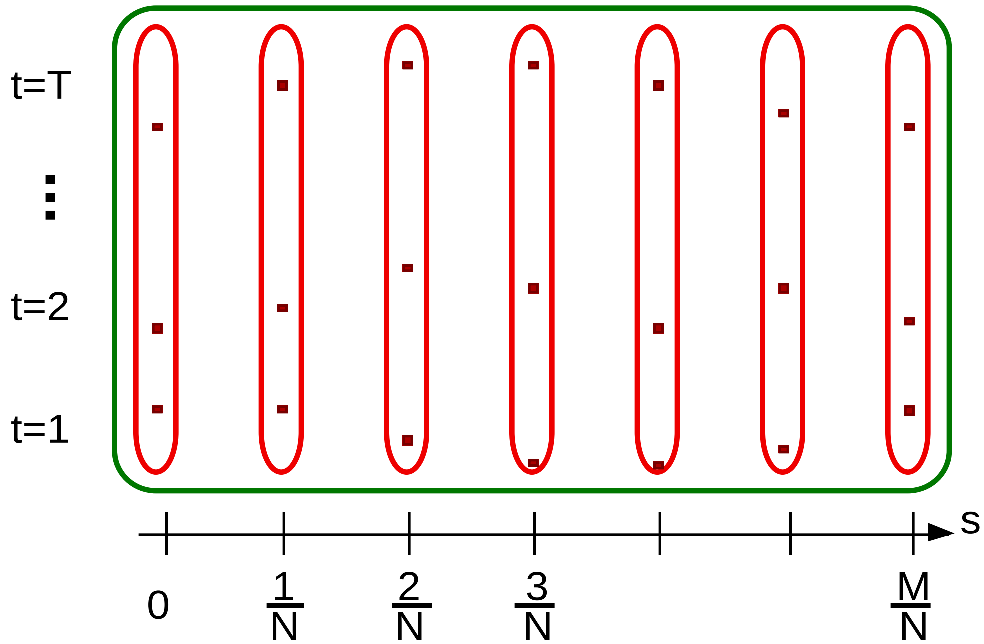

(1×T) encoding

(M×T) decoding

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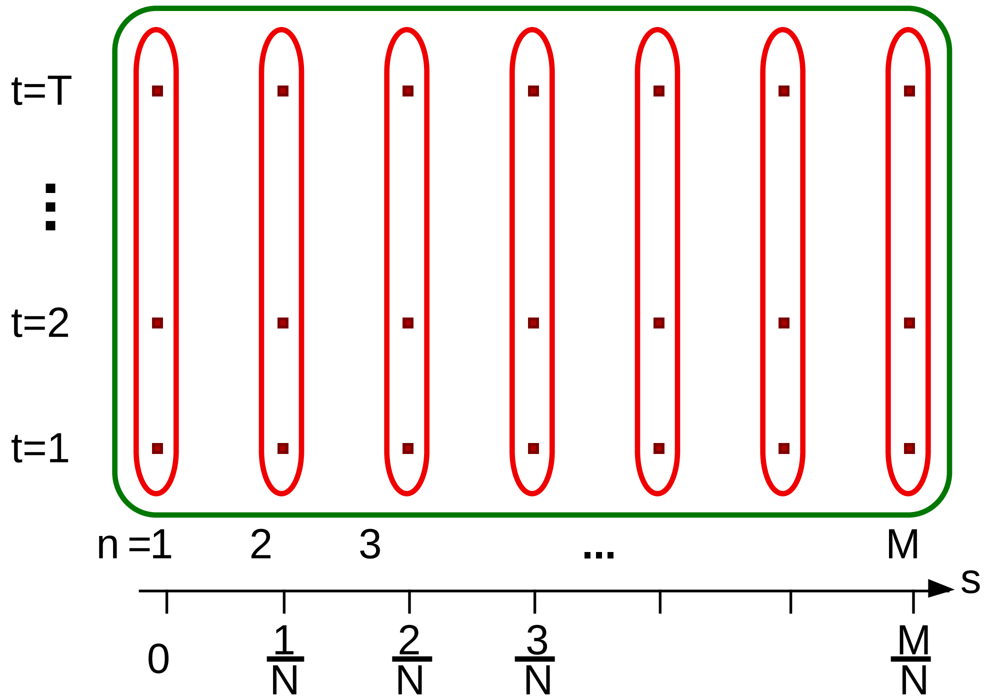

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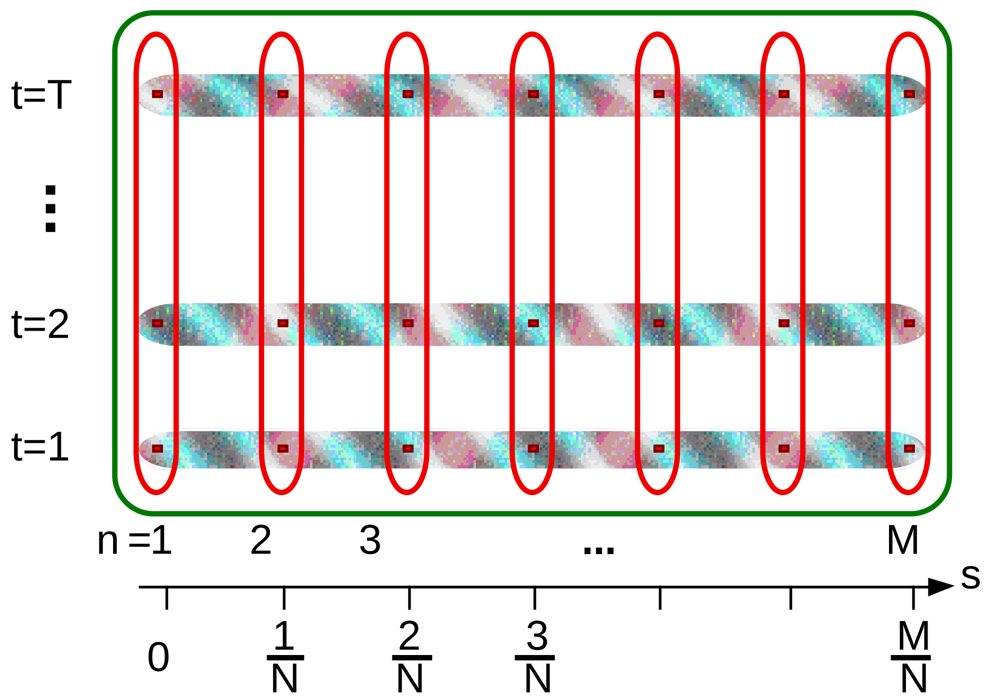

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Mean-Squared Error (MSE) Distortion

- For samples: $$D_N = \frac{1}{T}\sum_{t=1}^{T}\frac{1}{M}\sum_{n=1}^{M} E\left(Y_n^{(t)} - \hat{Y}_n^{(t)}\right)^2$$

- For cont.-space signal:

$$D_N = \frac{1}{T}\sum_{t=1}^{T}\frac{N}{M}\int_0^{M/N} E\left(X(t,s) - \hat{X}(t,s)\right)^2 ds \quad \cong D_N \quad \text{when } N \text{ large}$$

Optimum Performance

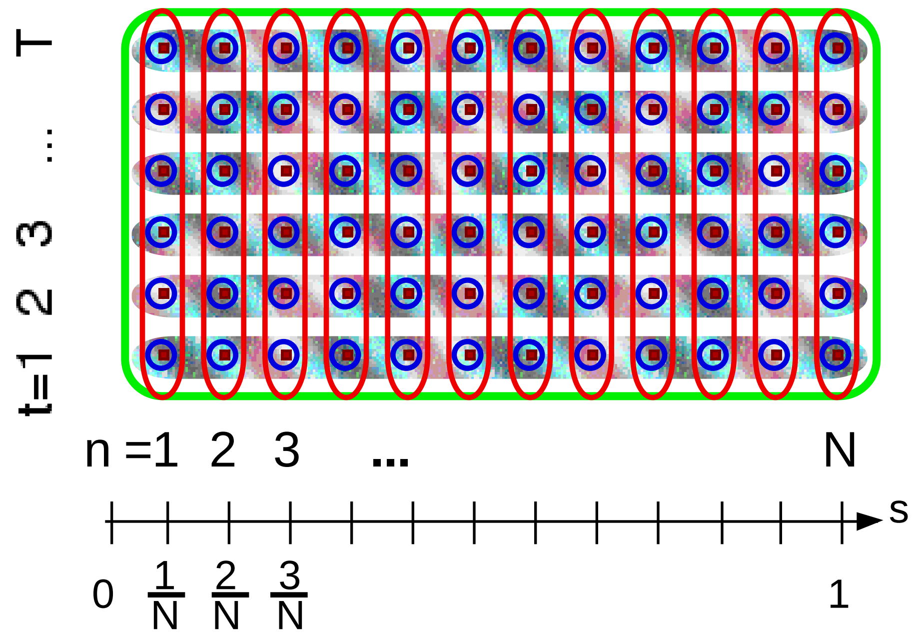
- For sampling rate N and for *given type of coding*

$$R_N(d) = \min_{D(\text{code}) \le d} R_N(\text{code}) \quad \text{bits/sample}$$

$$R_N(d) = N\, R_N(d) \quad \text{bits/meter}$$

$$R_\infty(d) = \lim_{N\to\infty} R_N(d) \quad \text{bits/meter}$$

1. MARCO ET AL. *(IPSN '03)*



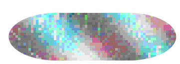
- Distributed Encoder
  - Each sample scalar quantized (1×1)

$$\hat{Y}_n^{(t)} = q(Y_n^{(t)})$$

  - Then T samples at each site encoded with Slepian-Wolf distributed lossless coding (1×T), T >> N = M

$$R_N = \frac{1}{N} H\left(\hat{Y}_1^{(t)}, \ldots, \hat{Y}_N^{(t)}\right) \quad \text{b/sample}$$

- **Slepian-Wolf decoder** (N×T)

- Linear interpolation: 

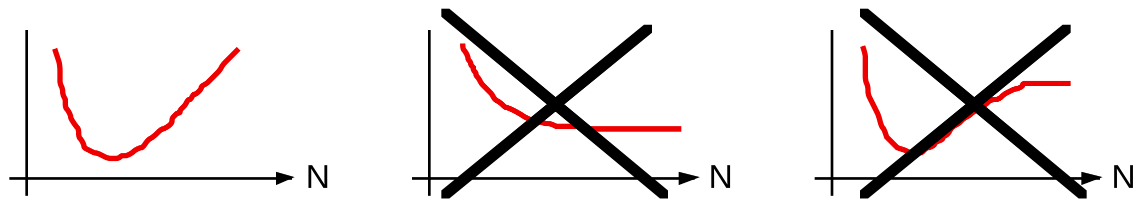
$$\hat{X}(t,s) = \sum_{n=-\infty}^{\infty} \hat{Y}_n^{(t)} \pi_N\left(s - \frac{n}{N}\right)$$

- **Theorem** -- For any stationary source X, any scalar quantizer q such that $d(q) < \sigma_X^2$, and any linear interpolation (which may depend on N),

$$\liminf_{N\to\infty} D_N > 0 \qquad \text{and} \qquad \liminf_{N\to\infty} R_N = \liminf_{N\to\infty} H(\hat{Y}_1^{(t)}, \ldots, \hat{Y}_N^{(t)}) = \infty$$

- Therefore, when using identical scalar quantization, ideal distributed lossless coding, and linear interpolation, distortion stays bounded away from zero and rate goes to infinity, and so

$$R_N(d) \to \infty \text{ as } N \to \infty$$



- Moreover,

$$\liminf_{N\to\infty} \inf_{q,\pi:\, D_N(q,\pi)\le d} H(\hat{Y}_1^{(t)}, \ldots, \hat{Y}_N^{(t)}) = \infty$$

$\Rightarrow$ adapting q to sampling rate N does not solve the problem.

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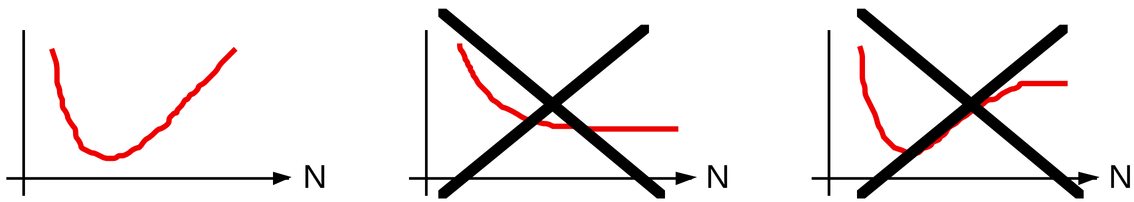


- Moreover,

$$\liminf_{N\to\infty} \; \inf_{q,\pi:\, D_N(q,\pi)\le d} H(\hat{Y}_1, \ldots, \hat{Y}_N) = \infty$$

$\Rightarrow$ adapting q to sampling rate N does not solve the problem.

Why Does $H(\hat{Y}_1, \ldots, \hat{Y}_N)$ Grow to ∞?

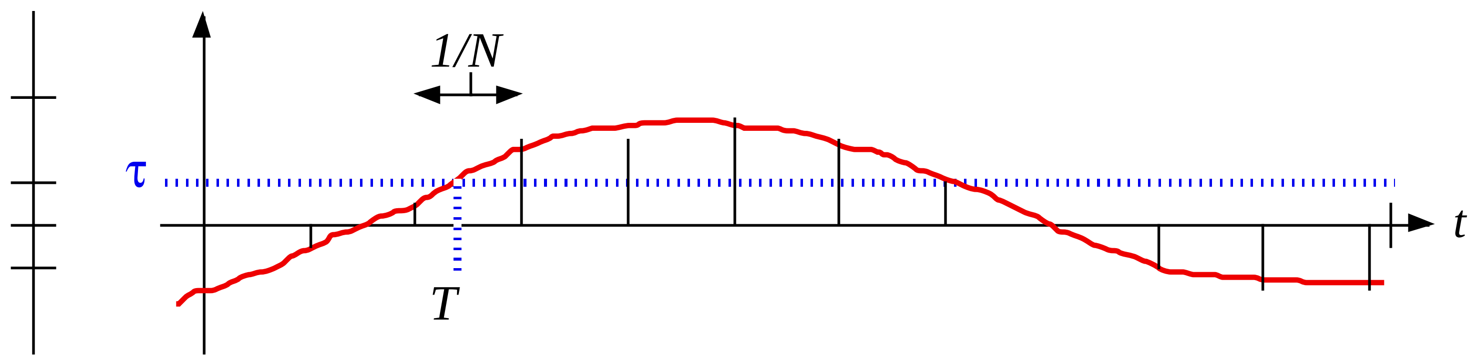
A scalar quantizer provides too much information!

As sampling rate increases, it provides enough information to determine zero crossings with arbitrary accuracy.

Crossing times are continuous or mixed random variables with infinite entropy.

They are determined with arbitrary accuracy from the sequence of quantized values.

Therefore, the number of bits needed to code the quantizer outputs, i.e. their entropy, goes to infinity.



At What Rate Does $H(\hat{Y}_1, \ldots, \hat{Y}_N)$ Grow to ∞?

Theorem: (ISIT 2005) For uniform scalar quantizer with infinitely many levels, step size Δ, and for stationary Gaussian $X(t)$, $N(0,1)$, autocorr. func. $\rho(\tau)$,

$$H(\hat{Y}_1, \ldots, \hat{Y}_N) \leq N\, H(\hat{Y}_2 | \hat{Y}_1) \cong -N m \sqrt{1-\rho(1/N)} \log_2 \sqrt{1-\rho(1/N)}$$

where

$$m = -\frac{2\sqrt{2}}{\pi} \sum_{k=0}^{\infty} e^{-\frac{(k+1/2)^2 \Delta^2}{2\sigma^2}}$$

Examples: When N is large

$$\rho(\tau) = e^{-|\tau|} \quad \Rightarrow \quad N\, H(\hat{Y}_2 | \hat{Y}_1) \cong \frac{m}{2} \sqrt{N} \log_2 N$$

$$\rho(\tau) = e^{-\tau^2} \quad \Rightarrow \quad N\, H(\hat{Y}_2 | \hat{Y}_1) \cong \frac{m}{2} \log_2 N$$

$$\rho(\tau) = \mathrm{sinc}(\tau) \quad \Rightarrow \quad N\, H(\hat{Y}_2 | \hat{Y}_1) \cong \frac{m}{2} c \log_2 N$$

SKETCH OF DERIVATION

$$H(\hat{Y}_2|\hat{Y}_1) = \sum_{k=-\infty}^{\infty} H(\hat{Y}_2|\hat{Y}_1=y_k)\, P_k \cong \sum_{|k|\le K(\rho)} H(\hat{Y}_2|\hat{Y}_1=y_k)\, P_k$$

where $K(\rho) = \left\lfloor \left(\ln \frac{1}{1-\rho(1/N)}\right)^{3/4} - \frac{1}{2} \right\rfloor$

Let $h(p) = -\,p \log_2 p$ and $P_{j|k} = \Pr(\hat{Y}_2=y_j|\hat{Y}_1=y_k)$. Then

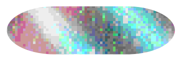
$$H(\hat{Y}_2|\hat{Y}_1=y_k) = \sum_{j=-\infty}^{\infty} h(P_{j|k}) \cong h(P_{k-1|k}) + h(P_{k|k}) + h(P_{k+1|k}), \quad |k|\le K(\rho)$$

$$\cong h(P_{k-1|k}) + h(P_{k+1|k}) \qquad \text{when} \quad \rho \cong 1$$

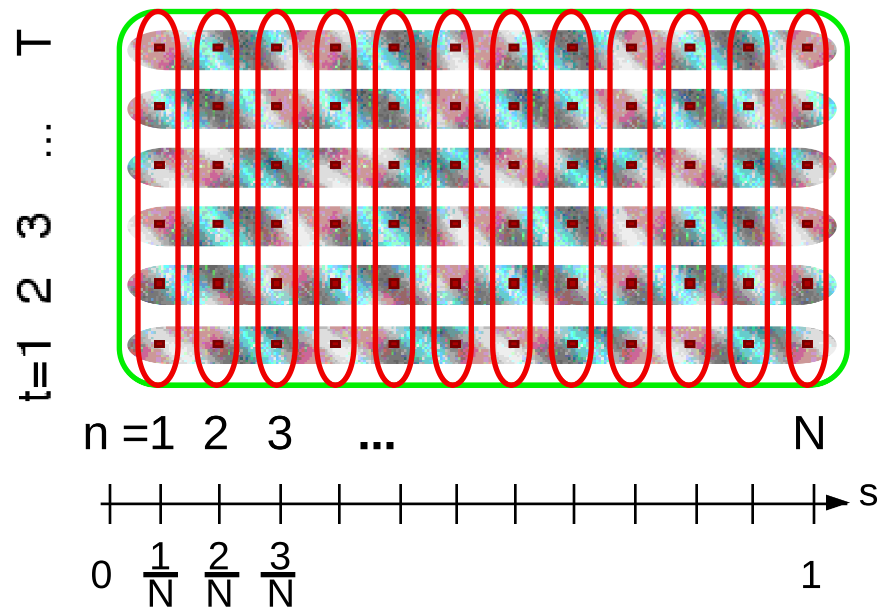
$$H(\hat{Y}_2|\hat{Y}_1) \cong \sum_{|k|\le K(\rho)} \left(h(P_{k-1|k}) + h(P_{k+1|k}) \right) P_k$$

$$\cong m \sqrt{1-\rho(1/N)} \log_2 \sqrt{1-\rho(1/N)}, \quad m = -\frac{2\sqrt{2}}{\pi} \sum_{k=0}^{\infty} e^{-\frac{(k+1/2)^2 \Delta^2}{2\sigma^2}}$$

2. KASHYAP ET AL *(DCC '05)*

- **Distributed encoder** $(1 \times T)$
- **Decoder** $(N \times T)$, $T >> N = M$
- Linear interpolation: 

$$\hat{X}(t,s) = E\left[X(t,s) \mid \hat{Y}^{(t)}_{[sN]/N}\right]$$



- **Theorem:** when N large

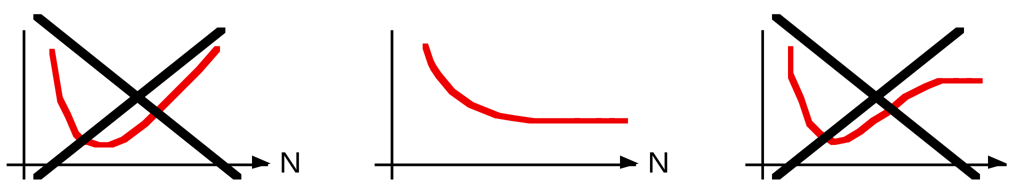
$$R_N(d) \le R^*_N(d) + \frac{N}{2}\log\left(1 + \frac{d}{p_N}\right)$$

$$\le R^*_N(d) + \text{constant} \le R^*_\infty(d) + \text{constant} < \infty$$

where

$$R^*_N(d) = \text{r vs d for opt'l centralized code, which is bounded}$$

$$p_N = \max\{p : \frac{1}{N} E\|\underline{Y} - E[\underline{Y}|\underline{Y}+\underline{Z}]\|^2 \le d,\ \underline{Z} = N(\underline{0}, pI)\}$$



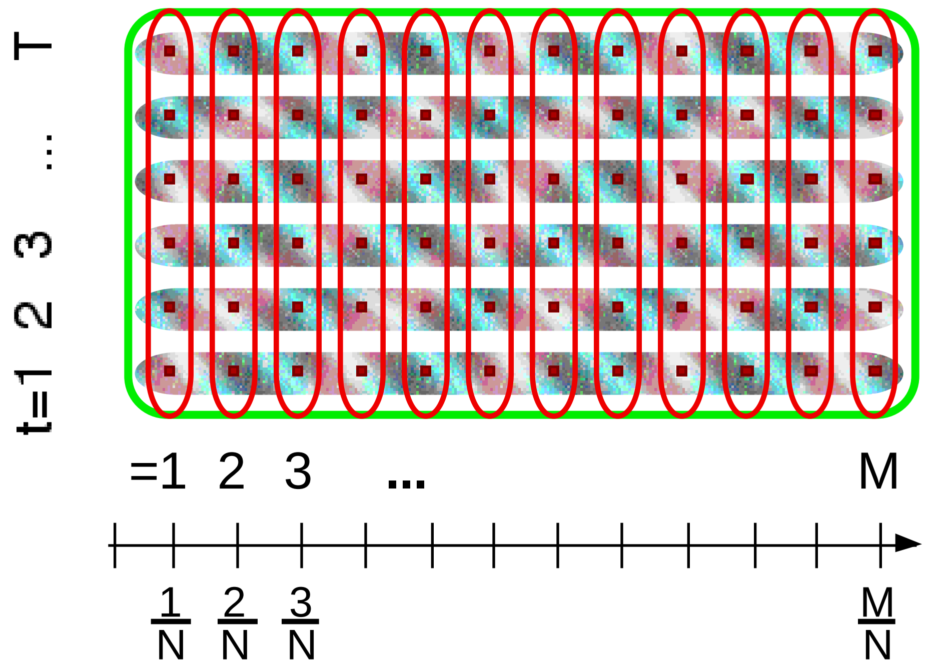
3. Neuhoff, Pradhan *(ICASSP '06)*

- Distributed Encoder (1×T)
- **Decoder** (M×T), T >> M >> N
- Linear interpolation:

$$\hat{X}(t,s) = \hat{Y}^{(t)}_{[sN]/N} \quad \text{(nearest } \hat{Y}\text{)}$$

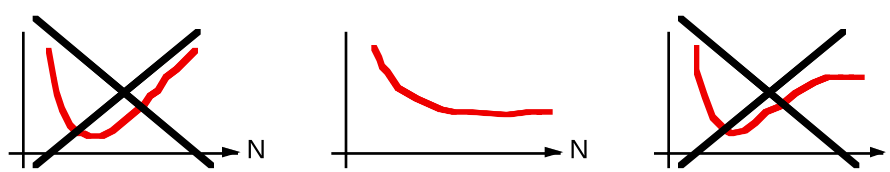
- Order of limits

$$\lim_{N\to\infty} \lim_{M\to\infty} \lim_{T\to\infty} D \text{ or } R$$



- **Theorem:** For a stat'ry, Gaussian cont.-space source with power spectral density $S(\Omega)$ and any $\theta > 0$, the following rate-distortion pair is attainable with distributed coding:

$$D(\theta) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \left(\frac{S(\Omega)}{\frac{S(\Omega)}{\theta} + 1} \right) d\theta \quad \text{and} \quad R(\theta) = \frac{1}{4\pi}\int_{-\infty}^{\infty} \log\left(\frac{S(\Omega)}{\theta} + 1 \right) d\theta$$



COMPARISON

Distributed Coding

(attainable inner bound)

$$D(\theta) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\frac{S(\Omega)}{\frac{S(\Omega)}{\theta}+1}\right)d\Omega$$

$$R(\theta) = \frac{1}{4\pi}\int_{-\infty}^{\infty}\log\left(\frac{S(\Omega)}{\theta}+1\right)d\Omega$$

Centralized Coding

(optimal)

$$D(\theta) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\min\{S(\Omega), \theta\}\, d\Omega$$

$$R(\theta) = \frac{1}{4\pi}\int_{-\infty}^{\infty}\max\{\log\frac{S(\Omega)}{\theta}, 0\}\, d\Omega$$

Kashyap et al.

$$R_N(d) \leq R_N^*(d) + \frac{N}{2}\log\left(1+\frac{d}{p_N}\right)$$

$$\cong R_N^*(d) + \text{constant}$$

$$\cong R_\infty^*(d) + \text{constant} \quad \text{when } N \text{ large}$$

EXAMPLE

- Source -- cont.-space, stationary, Gauss-Markov

  autocorr. funct. $e^{-|\tau|}$, pow. sp. dens. $S(\Omega) = \frac{2}{1+\Omega^2}$

Distributed Coding (attainable inner bound)

$$D(\theta) = \frac{1}{2}\left(\frac{\sqrt{\theta}}{\sqrt{\theta+2}}\right)$$

$$R(\theta) = \frac{1}{2\ln 2}\left(\frac{\sqrt{\theta+2}}{\sqrt{\theta}} - 1\right)$$

$$\Rightarrow R(d) = \frac{1}{2\ln 2}\left(\frac{1}{d} - 1\right)$$

Centralized Coding (optimal)

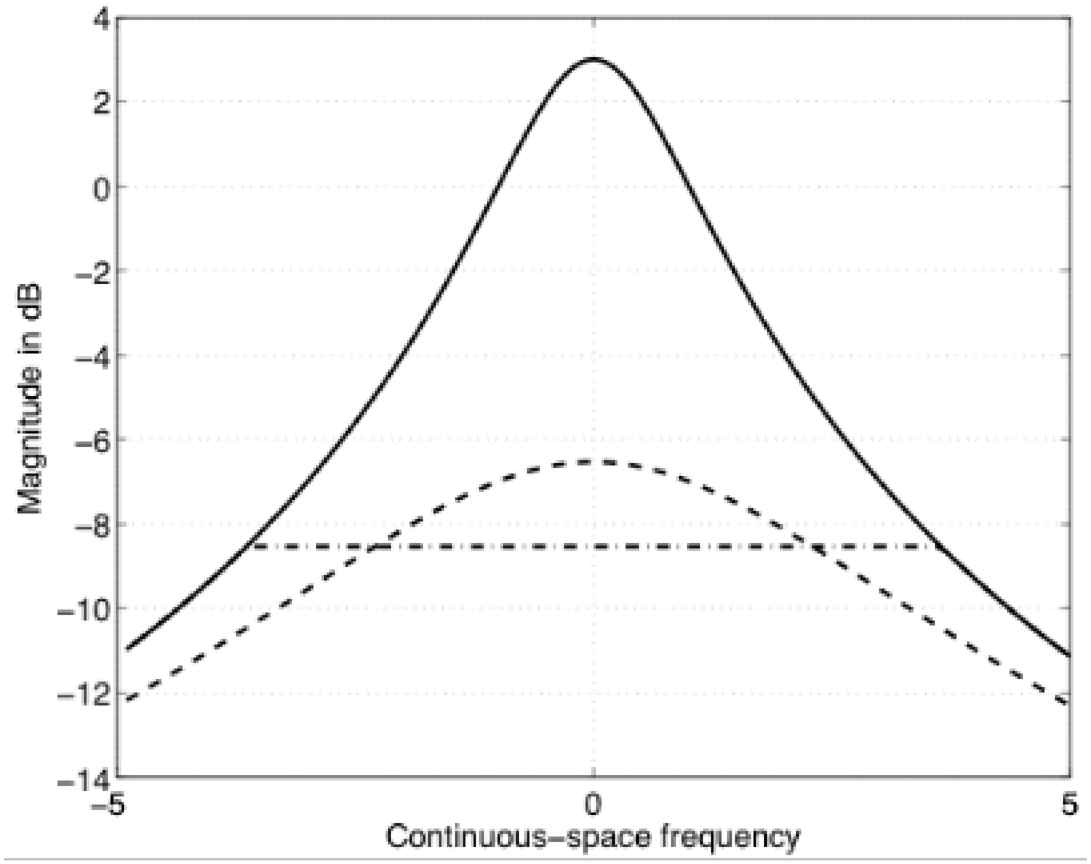
$$D(\theta) = 1 + \frac{1}{\pi}\sqrt{2\theta-\theta^2} - \frac{2}{\pi}\tan^{-1}\sqrt{\frac{2-\theta}{\theta}}$$

$$R(\theta) = \frac{1}{\pi}\left(\sqrt{\frac{2-\theta}{\theta}} - \tan^{-1}\sqrt{\frac{2-\theta}{\theta}}\right)$$

$$\Rightarrow R(d) \cong \frac{1}{2\ln 2}\left(\frac{.81}{d} - 1\right) \quad \text{for } d \cong 0$$

	Distributed Coding	Centralized Coding
@ d = 0.1,	6.5 b/m	5.1 b/m
Kashyap: N=100,	21.5 b/m	13.1 b/m

DISTORTION PROFILES



distributed

$$D(\theta) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\frac{S(\Omega)}{\frac{S(\Omega)}{\theta}+1}\right)d\Omega$$

centralized

$$D(\theta') = \frac{1}{2\pi}\int_{-\infty}^{\infty}\min\{S(\Omega),\theta'\}d\Omega$$

θ, θ' chosen so $D = 1/3$

EXAMPLE

- Source -- cont.-space, stationary, bandlimited, flat spectrum, Gaussian

$$S(\Omega) = \begin{cases} \pi/\Omega_o, & |\Omega| \leq \Omega_o \\ 0, & \text{else} \end{cases}$$

Distributed Coding

(attainable inner bound)

$$D(\theta) = \frac{\Omega_o \theta}{\Omega_o \theta + \pi}$$

$$R(\theta) = \frac{\Omega_o}{2\pi} \log \frac{\Omega_o \theta + \pi}{\Omega_o \theta}$$

$$\Rightarrow R(d) = \frac{\Omega_o}{2\pi} \log \frac{1}{d}$$

Centralized Coding

(optimal)

$$D(\theta) = 1 + \frac{1}{\pi}\sqrt{2\theta - \theta^2} - \frac{2}{\pi}\tan^{-1}\sqrt{\frac{2-\theta}{\theta}}$$

$$R(\theta) = \frac{1}{\pi}\sqrt{\frac{2-\theta}{\theta}} - \frac{1}{\pi}\tan^{-1}\sqrt{\frac{2-\theta}{\theta}}$$

$$\Rightarrow R(d) = \frac{\Omega_o}{2\pi} \log \frac{1}{d}$$

	Distributed Coding	Centralized Coding
@ $d = 0.1$, $\Omega_o = \pi$,	1.7 b/m	1.7 b/m
Kashyap: N=100	4.7 b/m	3.2 b/m

SKETCH OF PROOF

Steps

A. Berger-Tung attainable inner bound for coding samples $Y_n^{(t)}$
Berger *'77*, Berger-Zamir *'99*, Viswanath *'02*

B. Take $M \to \infty$ using Grenander-Szego asymptotic eigenvalue theorem.

C. Take sampling rate $N \to \infty$.

A. Berger-Tung Bound

- Given N, M, then for any $p(\underline{u}|\underline{y}) = \prod_{i=1}^{M} p(u_i|y_i)$ and all sufficiently large T, the following is attainable with $M \times T$ distributed coding:

$$D_{N,M} = \frac{1}{M} E \left\| \underline{Y} - E[\underline{Y}|\underline{U}] \right\|^2 \quad \text{and} \quad R_{N,M} = \frac{1}{M} I(\underline{Y};\underline{U})$$

where $\underline{Y} = (Y_1^{(t)}, ..., Y_M^{(t)})$, $\underline{U} = (U_1,...,U_M)$

- Our choice of $p(\underline{u}|\underline{y})$: for $\phi > 0$

$$U_i = Y_i + Z_i, \; i = 1,...,M, \text{ where } Z_i\text{'s are IID } N(0,\phi)$$

- After analysis, the following are attainable:

$$D_{N,M}(\phi) = \frac{1}{M} \sum_{i=1}^{M} \frac{\lambda_i}{\lambda_i/\phi + 1} \quad \text{and} \quad R_{N,M}(\phi) = \frac{1}{2M} \sum_{i=1}^{M} \log \left(\frac{\lambda_i}{\phi} + 1 \right)$$

where $\lambda_1,...,\lambda_M$ are eigenvalues of $M \times M$ covariance matrix for $\underline{Y}$

B. Grenander-Szego Theorem

Theorem: For any stationary discrete-space process Y with power spectral density $\Phi(\omega)$ and any piecewise continuous function g

$$\lim_{M\to\infty} \sum_{i=1}^{M} g(\lambda_i^{(M)}) = \frac{1}{2\pi} \int_{-\pi}^{\pi} g(\Phi(\omega))\, d\omega$$

It follows that for large M, T the following are attainable with a distributed $M \times T$ code

$$D_N(\phi) = \lim_{M\to\infty} \frac{1}{M} \sum_{i=1}^{M} \frac{\lambda_i}{\lambda_i/\phi + 1} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{\Phi(\omega)}{\Phi(\omega)/\phi + 1}\, d\omega$$

$$R_N(\phi) = \lim_{M\to\infty} \frac{1}{2M} \sum_{i=1}^{M} \log\left(\frac{\lambda_i}{\phi} + 1\right) = \frac{1}{4\pi} \int_{-\pi}^{\pi} \log\left(\frac{\Phi(\omega)}{\phi} + 1\right) d\omega$$

C. Sampling Rate $N \to \infty$

- Add N's to notation:

$$D_N(\phi_N) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{\Phi_N(\omega)}{\Phi_N(\omega)/\phi_N + 1}\, d\omega \quad \text{and} \quad R_N(\phi_N) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \log\left(\frac{\Phi_N(\omega)}{\phi_N}\right) d\omega$$

- Useful Fact: $\frac{1}{N}\Phi_N\left(\frac{\Omega}{N}\right) \to S(\Omega), \quad -\infty < \Omega < \infty$

- With goal of keeping $R_N(\phi_N)$ constant as $N \to \infty$, let $\phi_N = \theta n$.

- Then with change of variables -- $\Omega = \omega N$

$$D_N(\phi_N) = \frac{1}{2\pi}\int_{-\pi N}^{\pi N} \frac{\Phi_N(\Omega/N)/N}{\Phi(\Omega/N)/N\theta + 1}\, d\Omega \to \frac{1}{2\pi}\int_{-\infty}^{\infty} \left(\frac{S(\Omega)}{\frac{S(\Omega)}{\theta} + 1}\right) d\Omega \quad \text{bounded conv. thm}$$

$$R_N(\phi_N) = \frac{1}{4\pi}\int_{-\pi N}^{\pi N} \log\left(\frac{\Phi_N(\Omega/N)}{N\theta}\right)\frac{1}{N}\, d\Omega \to \frac{1}{4\pi}\int_{-\infty}^{\infty} \log\left(\frac{S(\Omega)}{\theta} + 1\right) d\Omega$$

DISCUSSION

- For a source with flat bandlimited power spectral density, optimal distributed coding is as good as optimal centralized coding, because

  sampling at Nyquist rate produces independent source samples,

  for which distributed coding is as good as centralized coding,

  and centralized coding at Nyquist rate is optimal

- For nonflat power spectral density, bandlimited or not, optimal distributed coding is not as good as optimal centralized coding, because

  sampling does not produce independent source samples,

  and the "effective" noise due to distributed coding is white followed by MMSE estimation,

  which cannot have the spectrum of the effective noise with optimal centralized coding,

  which is shaped according to inverse water pouring,

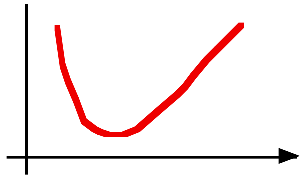
  and to attain such, requires the source samples to be KLT transformed, which is not permitted with distributed coding

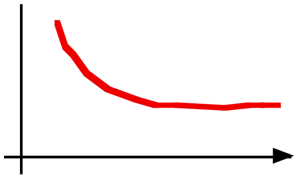
- Given recent proof by Wagner et al. that Berger-Tung is tight for two Gaussian sources, our asymptotic result may be tight.

CONCLUSIONS FOR OPTIMAL SYSTEMS

- Distributed source coding of a cont.-space Gaussian source is as good as optimal centralized coding for a flat bandlimited power spectral density, and not as good otherwise.
- Dense sensor networks can efficiently encode a stationary, Gaussian random field to any desired distortion d, in the sense that as sensor density increases, the number of bits/meter^2 remains finite, not far from the centralized rate-distortion function.
- Since Gaussian requires largest R for given D and power spectral density, a dense sensor network can efficiently encode an arbitrary stationary random field.
- Efficient encoding is possible with large N if $T \gg M \gg N$

  Not possible if quantizer dimension does not grow with N.
- If quantizer has fixed dimension, then for efficient encoding N must not be too large.





Comparison of Scalar Quantization-Based Results

- Mostly for bounded, deterministic, bandlimited signals,
  distortion $\rightarrow 0$ as $N \rightarrow \infty$

  a. Cvetkovic, Vetterli, *IEEE IT 1998, 2001*

  b. Cvetkovic, Daubechies, *DCC 2000*

  c. Ishwar, Kumar, Ramchandran, *IPSN 2003, 2004*

- For unbounded, random, bandlimited or not signals
  distortion fixed as $N \rightarrow \infty$

  1. Marco, Duarte-Melo, Liu, Neuhoff, *IPSN 2003*
     See also, Marco, Neuhoff, *ISIT 2005*
     distortion $\rightarrow 0$, rate $\rightarrow \infty$, as $N \rightarrow \infty$

SEVERAL DICHOTOMIES

bandlimited	vs.	nonbandlimited	signals
deterministic	vs.	random	signals
nonlinear	vs.	linear	reconstruction
code rate	vs.	entropy	as measure of rate
time-varying (dithered)	vs.	fixed	scalar quantizer
$D \to 0$	vs.	fixed	distortion constraint

Rate

- Bandlimited signals:

  In all known cases, rate increases with sampling rate N as

  $$\log N$$

- Nonbandlimited signals:

  For Gaussian source with autocorrelation function having nonzero slope at $\tau=0$,

  example: $\rho(\tau) = e^{-|\tau|}$

  an upper bound to rate for fixed scalar quantizers increases as

  $$\sqrt{N} \log N$$

  Conjectures:

  This is representative of actual rate, even for non Gaussian sources

  Same increase of rate for dithered quantizers

DISTORTION

- Linear reconstruction
  - Distortion does not go to zero as N increases, except in idiosyncratic cases

- Nonlinear reconstruction
  - Bandlimited, bounded sources

    Distortion can be made to go to zero.

  - Nonbandlimited, unbounded sources

    Whether or not distortion can go to zero depends on the source.

    There are stationary Gaussian sources for which it cannot *[Bar-David]*

    Conjecture: If a Gaussian source has strictly positive power spectral density, distortion cannot go to zero.

Conclusions for Scalar Quantizer Based Systems

- Bandlimited signals: The fact that scalar quantizer rate goes to infinity as $N \to \infty$ need not be so bad, because distortion can go to zero with nonlinear reconstruction. It might be possible to keep distortion and rate bounded as $N \to \infty$.

- There are some nonbandlimited signals for which distortion can go to zero and some where it cannot. The boundary is not clear.

  Conjecture: For most sources it does not.

  For sources where it does not, scalar quantization should not be used with high sampling rate

LAST WORDS

"Miss Kent, I want that researched, analyzed, verified, encoded, translated, extrapolated condensed, and typed in triplicate."