Decentralized Detection:

Tractability through Asymptotics

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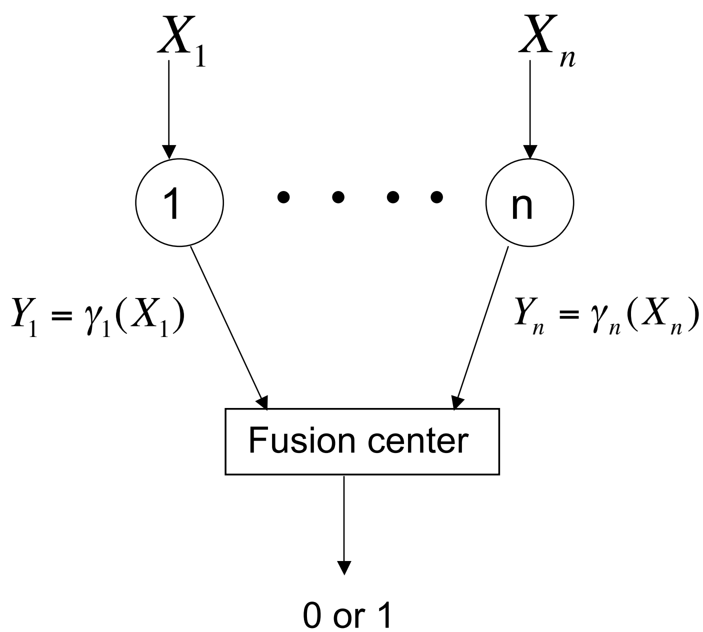
Outline

- Background
- A model with side information and censoring
- Neyman-Pearson formulation
- Bayesian formulation
- Discussion

The Classical Formulation (Tenney & Sandell, 1981)



- X_i are i.i.d.
- $H_0 : X_i \sim \mathbf{P}_0, \qquad H_1 : X_i \sim \mathbf{P}_1$
- $\gamma_i \in \Gamma$ e.g.: $\gamma : \mathbb{R} :\longrightarrow \{1, \ldots, m\}$
- minimize $\mathbf{P}(\text{error})$ over $\{\gamma_i\}$

- Likelihood ratio quantizers are optimal
  (compare $\mathbf{P}_0(X_i)/\mathbf{P}_1(X_i)$ to thresholds)
- Optimal thresholds can be different
- Without (conditional) independence of X_i: NP-hard (JNT+Athans, 85)

Asymptotics (JNT, 1988)

$$\begin{array}{rl} \text{minimize} & \mathbf{P}(\text{error} \mid H_1) \\ \text{subject to} & \mathbf{P}(\text{error} \mid H_0) \le \alpha \end{array}$$

$$\text{KL divergence :} \qquad I(\gamma) = \mathbb{E}_0 \left[\log \frac{\mathbf{P}_0(\gamma(X))}{\mathbf{P}_1(\gamma(X))} \right]$$

- Stein's lemma:

$$\mathbf{P}(\text{error} \mid H_1) \sim \exp\{-(I(\gamma_1) + \cdots + I(\gamma_n))\}$$

- Let γ maximize $I(\gamma)$
  - all sensors use that same γ
  - yields optimal exponent
  - randomization does not help

Side Information and Censoring

- S_i: side information (quality of measurement; quality of channel)
  - i.i.d.
  - $H_j : S_i \sim \mu_j(\cdot)$
  - $H_j : X_i \sim \nu_j(\cdot \mid S_i)$
- Censoring decision $\xi_i \in \{0, 1\}$ (Rago et al., Veeravalli, et al.)
- $\xi_i(S_i)$: no cooperation $\xi_i(S_1, \ldots, S_n)$: cooperation
- Transmission policy $\gamma_i \in \Gamma$ Message $Y_i = \gamma_i(X_i, S_i)$
- Fusion center knows all the S_i

Resource Constraints

- $\rho(S_i, \gamma_i)$: "consumption" of γ_i under "conditions" S_i

$$\overline{\rho}(\xi_i, \gamma_i) = \mathbb{E}_0\Big[\xi_i(S_1, ..., S_n)\, \rho(S_i, \gamma_i)\Big]$$

$$\frac{1}{n}\sum_{i=1}^{n} \overline{\rho}(\xi_i, \gamma_i) \leq c$$

- **Examples:**
  - $\rho(s, \gamma) = 1$
  - $\rho(s, \gamma) = \mathbb{E}_0\Big[|\gamma_i(X_i, S_i)|^2 \;\Big|\; S_i = s\Big]$

Randomization

- V random Strategy: $\pi_i : V \mapsto (\xi_i, \gamma_i)$
- Local π_i:
  - $\pi_i : V_i \mapsto (\xi_i, \gamma_i)$
  - V_i independent
  - $\xi_i : S_i \mapsto \{0, 1\}$ (no cooperation)
- Stationarity
  - V_i i.i.d.
  - π_i all the same
- Fusion center knows the V_i

Neyman-Pearson Results

If all use same, local $\pi = (\xi, \gamma)$: $\mathbf{P}_1(\text{error}) \sim \exp\{-n\lambda(\pi)\}$

$$\lambda(\pi) = \mathbb{E}_0\Big[\log \frac{d\mu_0}{d\mu_1}(S)\Big] + \mathbb{E}_0\big[\xi(S) I(S,\gamma)\big]$$

$$I(s,\gamma) = \mathbb{E}_0\Big[\log \frac{d\mathbf{P}_0}{d\mathbf{P}_1}(Y) \,\Big|\, S = s\Big], \qquad \text{where } Y = \gamma(X, s)$$

- Can achieve $\mathbf{P}_1(\text{error}) \sim \exp\{-n\lambda^*(c)\}$

$$\begin{aligned} \lambda^*(c) \;=\; & \sup \;\; \lambda(\pi) \\ & \text{s.t. } \; \mathbb{E}_0[\xi(S)\rho(S,\gamma)] \le c \end{aligned}$$

- Need randomization between at most 2 deterministic policies

Neyman-Pearson Lower Bound

$$-\frac{\lambda^*(c)}{1-\alpha} \;\le\; \liminf_{n\to\infty} \frac{1}{n} \log \mathbf{P}_1(\text{error}) \;\le\; -\lambda^*(c)$$

- If $\alpha \approx 0$: cooperation does not help
- w/o cooperation within $O(\alpha)$ from optimal
- For fixed α, cooperation helps
- **Example:**
  All "sleep" with probability $\alpha/2$; declare H_1
  Saves fraction $\alpha/2$ of resources
  As if larger c when "awake"

Discussion of Neyman-Pearson Case

$$\begin{aligned} &\sup_{\xi,\gamma} \ \mathbb{E}_0[\xi(S)I(S,\gamma)] \\ &\text{s.t. } \mathbb{E}_0[\xi(S)\rho(S,\gamma)] \leq c \end{aligned}$$

- Censor at those s for which $\dfrac{I(s,\gamma)}{\rho(s,\gamma)}$ is small

- Randomization used to make the constraint tight

Bayesian Problem w/o Cooperation

Classical error exponent: $\sum_{i=1}^{n} \Phi(\pi_i)$

$$\Phi(\pi) = \min_{s \in [0,1]} \log \mathbb{E}\left[\mathbf{P}_1^s(Y)\mathbf{P}_0^{1-s}(Y)\right]$$

$$\begin{array}{rl} \text{minimize} & \Phi(\pi) \\ \text{subject to} & \rho(\pi) \leq c \end{array}$$

- achieved with sensors divided into groups
  - each group uses same deterministic policy
- Best possible w/o cooperation

Cooperation Helps!

Constraint: Use only $n/4$ sensors, on average

$\mathbf{P}(S_i = 0) = 1/2$: useless measurement X_i
$\mathbf{P}(S_i = 1) = 1/2$: useful measurement X_i

Cooperative strategy: Use $n/4$ "good" sensors: $\exp\{-n\lambda/4\}$

Local strategy:

- $n/2$ sensors: sleep
- $n/2$ sensors: transmit if "good"
- N sensors transmitting, $\mathbb{E}[N] = n/4$
- $\mathbf{P}(\text{error}) \sim \mathbb{E}[e^{-N\lambda}] > e^{-\mathbb{E}[N]\lambda} = e^{-n\lambda/4}$

Discussion

- Intractable problems w/o asymptotics
- Emphasis on error exponents yields:
  - analytical solutions in simple settings
  - computable solutions in various examples
  - qualitative insights
- Curious dichotomy regarding “decentralization”:
  - Neyman-Pearson: does not hurt
  - Bayesian: it hurts
- Future: compare different architectures