Energy Efficient Policies for Distributed Target Tracking in Multihop Sensor Networks *

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Abstract

We consider the problem of distributed target tracking in a sensor network under communication constraints between the sensor nodes, a problem that has recently received significant attention. The problem requires the dynamic selection of which sensor nodes will communicate their information and the selection of a corresponding fusion center which will process the collected information. Ideally, selection of which sensors will communicate and where will fusion take place will be a dynamic process, adapting to new information, to trade off tracking accuracy versus communications usage. The resulting coupled problem is generally intractable and significant effort has been devoted towards proposing simple strategies under various performance criteria. In this paper, we propose an adaptive dynamic strategy for sensor selection and fusion location using a certainty equivalence approach that seeks to optimize a tradeoff between tracking error and communications cost We define a certainty equivalent optimization problem for dynamic relocation of the fusion center that uses measures of average multi hop communications cost and average tracking errors, and solve the resulting optimal control problem for classes of tracking problems. The optimal strategy is a hybrid switching strategy, where the fusion center location and reporting sensors are held stationary unless the target estimates move outside of a threshold radius around the sensors. We illustrate the performance of our algorithms on sample tracking experiments with sensor networks.

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1 Introduction

Consider the problem of tracking a target moving in region populated by sensor nodes that have limited wireless communication capabilities. The sensors have limited sensing range and the quality of measurements degrades with distance from the target. In such networks, the ideal tracking system would process measurements from every sensor at a single location to estimate the target state However, this centralization is not possible due to communication constraints on individual sensors.

In the presence of communication constraints, one must use a distributed mechanism for selecting which information should be communicated, and where should the information be processed. In this paper, we explore a particular distributed protocol for tracking in sensor networks with communication constraints. We assume that there is no central authority governing network operation. Since measurement quality degrades with distance, only sensors sufficiently close to the target should share their data. Furthermore, since there is no designated central authority an arbitrary sensor is designated as the fusion center (or *leader node*) for a given time period and the data is processed at this sensor. Therefore, the entire mechanism can be broken down into three tasks:

1. Given the current sensor set, fusion center and past target information, the fusion center estimates the current target position based on sensed information from sensor set.
2. Fusion center computes the new sensor set as well as the new fusion center for the next time period based on current estimated target state.
3. Fusion center then communicates state information to the new fusion center.

Communication costs arise while transmitting data from sensors to the fusion center and while transmitting current state information to a subsequent fusion center. Therefore, the fusion center should be close to the informative sensors and the distance from a fusion center to the next fusion center should be small. For accurate tracking the sensors should be selected sufficiently close to the target. This implies that for rapidly evolving trajectories the active sensor locations will need to move rapidly as well. This increases communication costs, so the problem is to find strategies that balance communication costs with accuracy.

One of the earlier approaches in the direction of finding the best informative sensor along with communication costs, at each time step, was taken in [1],[2]. In [3] non-myopic strategies for sensor selection was proposed with K-L distance as the information utility measure. However the cost of communication between the successive sensors and cost of aggregation was not taken into account. In [4] the problem of tracking in a similar framework was formulated as a dynamic program subjected to total communication constraints with information theoretic utility measures as cost per stage. Due to analytical intractability they used approximate dynamic programming methods, namely Lagrangian relaxation to solve finite horizon DP.

The problem of target tracking in adaptive sensor networks can be viewed as a stochastic control problem, where the state of the system corresponds to the individual target state, the reporting sensor set, and the location of the fusion center. The state of the system is partially observed, as the individual target state is observed through noisy sensors. The control actions belong to a discrete set, identifying which sensors will report measurements and where the fusion center is located. Hence, the overall problem can be modeled as a partially observed Markovian decision problem [5] with a combinatorial decision space and a large state space (with part of it continuous-valued). Solution of such problems is intractable even in small instances, and too complex for real time adaptation of sensor networks.

In contrast, we propose an alternative tractable formulation that retains essential features of the original problem: the inherent tradeoffs between the communication costs and the estimation performance in such distributed tracking problems. We develop a fully observed Markov decision problem formulation for the problem of sensor selection and fusion center selection. This formulation can be analyzed and solved for the feedback control policy that jointly optimizes measures of average multi-hop communications and average tracking error. We show that, for classes of target dynamic models, the optimal strategy exhibits a hybrid switching strategy characterized by dead zones. The sensor locations are stationary until the target exits the dead zone and then the locations are switched to best sensor locations around the target.

In order to apply our results to the control of sensor networks with partial information about target states, we adopt a certainty equivalent perspective, where target state estimates are treated as corresponding to actual current states. We implement our sensor network control algorithms in several tracking scenarios, and show the resulting reductions in communication that arise from effective network control in contrast to alternative algorithms.

The paper is organized as follows. In section 2 we will present the target tracking set-up and outline the general problem. We present in some detail the current difficulties with existing approaches. In section 3 we will formulate an analytically tractable abstraction for the tracking problem. In sections 4.1 and 4.2 we solve the abstracted problem for two cases of squared communication cost and L1 communication cost respectively. We propose solutions for several cases of target dynamics. Finally in section 7 we will present simulation results to see the performance.

2 General Problem

Consider a collection, $\mathcal{S}$ of n sensors at positions, s_1, $s_2, \ldots, s_n$. At any time k the observation at the sensor s_i is given by,

$$Z_k^i = F(X_k, s_i) + V_k^i$$

where X_k is the state at time k of the target, typically consisting of position and velocity, and V_k^i is additive additive, white Gaussian noise of variance Σ_v and is independent of $X_{k'}$ for all k'. The target state evolves as an autonomous system as follows:

$$X_{k+1} = f(X_k, k) + w_k \tag{1}$$

where w_k is zero mean white Gaussian noise of variance Σ_w which is assumed independent of $V_{k'}^i$ for all i, k' and independent of the initial condition X_0.

The problem of interest is to select the fusion center at time $k+1$ and the set of active sensors to report at time $k+1$, based on the information available at time k, with a goal of minimizing a tradeoff between communications cost and tracking error. A similar problem was studied in [4], where they sought to maximize the information collected over time subject to a communications constraint. The resulting formulation was a Partially Observed Markov Decision problem with a combinatorial decision space whose solution was intractable.

In this paper, we focus on the problem of locating the fusion center, and assume that the active sensors in a pre-specified neighborhood of the fusion center will be the reporting sensors. This reduces the combinatorial complexity of the decision problem while maintaining the fundamental issue of where to locate the fusion center to trade-off information as well as minimize communications. We assume that the fusion center is located at one of the sensor locations s_i, which also specifies a neighborhood N_i of sensors that will report observations.

Let ℓ_k denote the location of the fusion center at stage k. Let $\mathcal{I}_k$ denote the information available for decision at time k, which includes all of the past observations collected and the past locations of the fusion center, including the observations at time k. The control problem at time k is to select a strategy $\mu_k : \mathcal{I}_k \rightarrow \mathcal{S}$ that determines where the fusion center will be located at time $k-1$.

The performance objective for selecting a control strategy is a tradeoff between communications costs and tracking error. We express this cost over a finite horizon N

$$E\{\sum_{k=0}^{N-1}[Kc_1(\ell_{k+1}-\ell_k)+c_2(X_k,I_k)]+c_2(X_N,I_N)\} \tag{2}$$

where $c_1(\cdot)$ is the communications cost of switching the fusion center between locations, $c_2(\cdot,\cdot)$ represents the tracking error. For our purposes, we choose the square estimation error

$$c_2(X_k,I_k)=(X_k-E[X_k|I_k])^T Q(X_k-E[X_k|I_k])$$

where Q is a weighting matrix that can be used to select the position entries in X_k, or any other desired weighting. The communications cost is chosen as

$$c_1(u)=\parallel u \parallel_2^2 \text{ or } \parallel u \parallel_1$$

Note that, when $N>1$, the resulting problem is a partially observed Markov decision problem with an underlying continuous state space (X_k), rendering the problem intractable unless the sensor observations have special structure (e.g. linear Gaussian measurements or finite valued measurements), as the admissible control strategies functions of information sets that lived in continuous spaces, with no finite-dimensional sufficient statistics [5].

For the special case that the horizon $N = 1$, the problem reduced to evaluating the cost in (2) for each of the possible choices of fusion center locations at the next stage, $\ell_1 \in \mathcal{S}$. This can be done approximately using a Cramer-Rao bound approach or an extended Kalman filter (EKF) to approximate the error covariance of the estimate, as in [**?**], as follows. Let $\Sigma_{0|0}$ denote the error covariance $E[(X_0 - E[X_0|I_0])(X_0 - E[X_0|I_0)^T|I_0]$, and let $\hat{X}_0$ denote the estimate of the state at time 0 given I_0. Then, the EKF can be used to estimate the tracking error at time 1 given the choice of $\ell_1 = s_i$, as

$$\begin{aligned}
\hat{X}_{1|0} &= f(\hat{X}_0, 0) \\
\Sigma_{1|0} &= \frac{\partial}{\partial X} f(X,0)|_{\hat{X}_0} \Sigma_0 \frac{\partial}{\partial X} f(X,0)|_{\hat{X}_0}^T + \Sigma_w \\
C^j &= \frac{\partial}{\partial X} F(X, s_j)|_{\hat{X}_{1|0}} \\
\Sigma_{1|1}^{-1}(i) &= \Sigma_{1|0}^{-1} + \sum_{j \in N_i} (C^j)^T \Sigma_v^{-1} C^j \\
E[c_2(X_1, I_1)] &\approx Trace[\Sigma_{1|1}(i) Q] \qquad (3)
\end{aligned}$$

The choice of ℓ_1 is selected as

$$\begin{aligned}
i^* &= \mathrm{arg} min_i \{Trace[\Sigma_{1|1}(i)Q] + Kc_1(s_i - s_{i_0}) \\
\ell_i^* &= s_{i^*}
\end{aligned}$$

The above algorithm can be used in a receding horizon manner to generate a decision policy $\mu_k(I_k)$ for each time k, based on a one-step lookahead horizon. However, extending the approach to a policy that is based on looking ahead for more than one time period is computationally infeasible. In the next section, we present an approximate approach that allows us to develop decision policies based on multi-period horizons.

3 Certainty Equivalence Perspective

To get insight into the fundamental tradeoffs between communication and estimation performance we formulate a simpler problem, which nevertheless has the essential features of the original problem. We adopt the following strategy with the accompanying assumptions:

1. We adopt a certainty equivalence perspective and pose an analytically tractable problem for target tracking. In particular, we de-couple state estimation and control, which reduces the complexity of Task 2.

2. We assume a uniform placement of sensor nodes in the sensing region. This implies the set of most informative sensors is typically a small subset of sensors around a small radius around the target position.

3. The quality of observations degrades substantially with increasing sensor distance from the target.

4. The communications cost can be approximated by distance of the fusion center to the target. This follows because of two equivalent possibilities: (a) fusion center is not updated but the sensor set is updated to get information close to the target, (b) both fusion center and sensor set are close to target. In the former case although there is no communication penalty for communicating past information, there is a high cost to communicating sensor information. Since the number of active sensors at any time is a constant this cost is a constant multiple of scenario (b).

Multi-hop Communication Costs & ℓ_1 penalty Although communication energy is directly proportional to the square (or fourth power) of the distance of communication, this penalty is not truly reflective of meshed networks where multi-hop costs dominate. In a low power sensor network where the *communication protocol* is restricted to be multi-hop, the communication cost is dominated by the number of hops taken from the source to destination, in this case from current leader node to the next one. This means that it depends mainly on the absolute distance (that determines the number of hops) and not on the squared distance between the nodes. Thus ℓ_1 norm on the communication distance appears to account for communication in a multi-hop network. Multi-hop operation leads to significant reduction in communication energy. This is due to the fact that the energy requirements for multi-hop operation is proportional to number of hops taken, whereas for direct communication the energy requirements are proportional to d^{α} where α is the attenuation factor,($\alpha = 2$) for free space). This essentially amounts to significantly larger communication energy as illustrated in Figure 1.

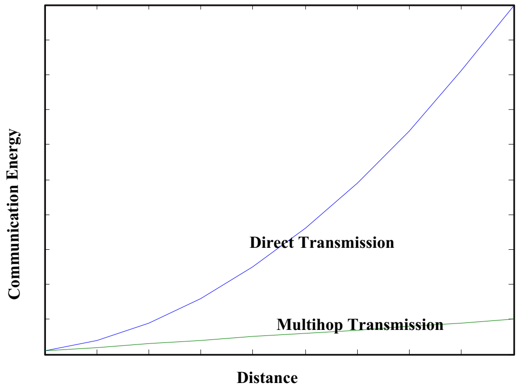


Figure 1: Illustrates tradeoff between communication energy with varying distances for direct transmission and multi-hop schemes in a regular mesh sensor network where the spacing between any two nodes equals one.

To simplify our analysis we assume a continuum of sensors, i.e., there are sensors everywhere at any point in the region. We identify each sensor with its location. With slight abuse of notation a leader node will be denoted by $l(t)$ which also denotes the location of the sensor. The error in state estimation is inversely proportional to the power received, and is thus proportional to $||x(t)-l(t)||^2$ for a free space wave propagation model. The decision variable at time t is the location of the next active sensor, denoted by $l(t+1)$, based on the current position $x(t)$ of the target. Let $u(t) = l(t+1) - l(t)$. Then the cost of communication is proportional to $||u(t)||_1$ for multi-hop networks and proportional to $||u(t)||^2$ for direct transmission in free space. Assuming the proportionality constants to be unity we have the following two candidate unconstrained optimization problems.

$$Q1 : \min_{u(1),..,u(N-1)} \mathbf{E} \sum_{t=1}^{N-1} ||u(t)||_1 + ||x(t) - l(t)||^2 + ||x(N) - l(N)||^2$$

$$Q2 : \min_{u(1),..,u(N-1)} \mathbf{E} \sum_{t=1}^{N-1} ||u(t)||^2 + ||x(t) - l(t)||^2 + ||x(N) - l(N)||^2$$

where the expectation is over the process noise in the target dynamics. These two simple models assume that there is continuum of sensors and that the state of the target is noiselessly observable.

4 Random Walk

In the following we will present analysis for one dimensional discrete target dynamics. The results extend to the two dimensional case in a straightforward way as the minimization at each step is decoupled in different dimensions. Let the target dynamics (in one dimension) be given by

$$x_{k+1} = x_k + w_k$$

where k is the discrete time index and w_k is a zero mean noise independent of x_k. If w_k is zero mean Gaussian random variable, then x_k is the standard Random Walk starting at zero. Let the state of the system (sensor network) be given by the $\tilde{x}_k = [x_k, l_k]'$ i.e. the current state of the target and the current activated node. Then we have the following dynamics for the extended state space.

$$\tilde{x}_k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tilde{x}_k + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_k + \begin{bmatrix} 1 \\ 0 \end{bmatrix} w_k \tag{4}$$

Effectively the state of the system can also be captured by $z_k = x_k - l_k$. Then the system equation evolves according to the function

$$z_{k+1} = z_k - u_k + w_k$$

Under the above simple dynamics we have the following optimization problems to solve.

$$Q1 : \min_{u_1,..,u_{N-1}} \mathbf{E} \left(\sum_{k=1}^{N-1} \left[||u_k||_1 + ||z_k||^2 \right] + ||z_N||^2 \right)$$

$$Q2 : \min_{u_k,.,u_{N-1}} \mathbf{E} \left(\sum_{k=1}^{N-1} \left[||u_k||^2 + ||z_k||^2 \right] + ||z_N||^2 \right)$$

4.1 Policy under ℓ_2 Communication Model

We consider first the case of quadratic penalty on the control action, i.e., communication cost. Under the state dynamics model for z_k the optimization problem $Q2$ is the standard LQ-controller, under perfect state observation, the solution to which is known, [5]. At any time k the optimal control action is of the form $u_k = -L_k z_k$. Notice that if z_k is a random walk in the absence of control, then the optimal control is a scaled random walk. This also implies that one has to *switch* to a new leader node all the time. Under multi-hop operation the communication cost—which amounts to the total variation of the scaled random walk—goes to infinity. Thus for multi-hop networks an ℓ_2 constraint does not reflect the true communication costs.

4.2 Policy under ℓ_1 Communication penalty (Multi-hop)

To account for multi-hop communication costs we penalize communication costs through an ℓ_1 cost criterion. For sake of exposition we consider two cases, viz., (a) no process noise, i.e., a stationary target and (b) a uniform process noise.

4.2.1 No Noise Case

For no process noise, i.e. $w_k = 0$, and perfect state observation, consider the cost to go at time $N-1$,

$$J_{N-1}(x_{N-1}) = \min_{u_{N-1}} \left\{ |u_{N-1}| + z_{N-1}^2 + (z_{N-1} - u_{N-1})^2 \right\}$$

The derivative of $|u_{N-1}|$ is not defined at zero. For this we will find the subgradient set of the function inside the minimization. At zero the subgradient set of $|u_{N-1}|$ is given by $[-1, 1]$. Thus if $|z_{N-1}| < 1/2$ then zero is an element of the subgradient set. This implies that optimal control $u_{N-1} = 0$ if $|z_{N-1}| < 1/2$. Then the optimal cost to go at stage $N-1$ is given by

$$J_{N-1}(z_{N-1}) = \begin{cases} 2z_{N-1}^2 & \text{if } |z_{N-1}| \leq 1/2 \\ |z_{N-1}| - 1/4 + z_{N-1}^2 & \text{if } |z_{N-1}| > 1/2 \end{cases}$$

The interpretation of the control at N-1 is that we do not apply any control if the state z_{N-1} is in a "dead-zone" region $[-1/2, 1/2]$.

It can be shown that at time $N-k$ the cost to go function is given by

$$J_{N-k}(z_{N-k})$$

$$= \begin{cases} (k+1)z_{N-k}^2 & |z_{N-k}| \leq 1/2k \\ |z_{N-k}| - \frac{1}{2k} + z_{N-k}^2 + \frac{k}{4k^2} & |z_{N-k}| > 1/2k \end{cases}$$

and the optimal "dead-zone" region at stage $N-k$ is given by $[-1/2k, 1/2k]$. Note that the cost to go is a smooth convex function. The dead-zone region shrinks. This is reflective of the fact that at any stage if the state $|z_k| = |x_k - l_k|$ is below a certain threshold then no control is applied. This result can be used to effectively localize a stationary target with minimal communication costs in a multi-hop network.

4.2.2 Uniform Bounded Process Noise

Suppose now that the noise process w_k is not zero. For the sake of analytical tractability we assume that the noise process w_k is a uniform noise bounded between $[-\alpha, \alpha]$. Under perfect state observation in this case we have for cost to go at stage $N-1$,

$$J_{N-1}(z_{N-1}) = \min_{u_{N-1}} \left\{ \lambda |u_{N-1}| + z_{N-1}^2 + \mathbf{E}_w (z_{N-1} - u_{N-1} + w_{N-1})^2 \right\}$$

Since the noise is zero mean and uncorrelated, the cost to go at stage $N-1$ is given by,

$$J_{N-1}(z_{N-1}) = \begin{cases} 2z_{N-1}^2 + \sigma_w^2 & |z_{N-1}| \leq 1/2 \\ |z_{N-1}| - 1/4 + z_{N-1}^2 + \sigma_w^2 & |z_{N-1}| > 1/2 \end{cases}$$

We now have the following surprising result.

Theorem 4.1. *The optimal n-stage policy is a switching policy, i.e.,*

$$\begin{aligned} u_n &= 0, \; |z| \leq \Gamma_n \\ u_n &= (|z| - \Gamma_n) sign(z), \; |z| \geq \Gamma_n \end{aligned}$$

where, $\Gamma_n > 0$*. The corresponding n-stage cost-to-go is described by:*

$$J_n(z) = \begin{cases} z^2 + \mathbf{E}(J_{n-1}(z+w)) & |z| \leq \Gamma_n \\ \lambda|z| - \Gamma_n + z^2 + \mathbf{E}J_{n-1}(\Gamma_n + w) & |z| > \Gamma_n \end{cases}$$

The switching point Γ_n *is uniformly bounded from below, i.e.,*

$$\Gamma_n \geq \Gamma_0 = \min\left(\frac{\alpha}{2}, \frac{\alpha\lambda}{\alpha+\lambda}\right)$$

Consequently, the infinite horizon policy is a switching policy as well.

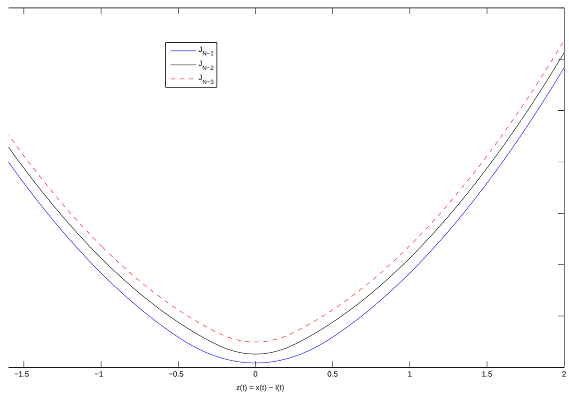
Proof. See Appendix. □

For purpose of illustration, we choose a uniform noise between $[-0.25, 0.25]$. The optimal cost to go for various stages and the optimal control at various stages are shown in figure 2.

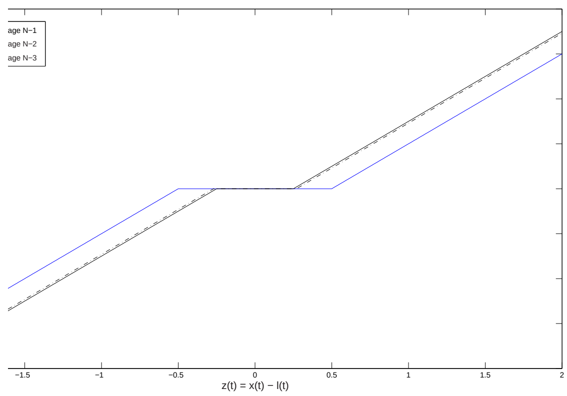
This implies that the optimal stationary policy is described by a footprint region (or a deadbeat zone) corresponding to each sensor node. The processing is localized at a node as long as the target remains within that nodes footprint and the processing center is switched as soon as the target leaves this region. Observe that this strategy is specific to multi-hop networks due to the ℓ_1 penalty on communication costs.

5 Random Walk with Drift

Now consider the target motion described by the following dynamical equation.



(a)



(b)

Figure 2: (a) The optimal cost to go for three stages in the backward DP recursion. (b) The optimal control policy as a function of the state $z(t)$ for various stages of DP recursion. Notice that the dead zone region in this case does not shrink to zero.

$$\begin{bmatrix} x(k+1) \\ v_x(k+1) \\ y(k+1) \\ v_y(k+1) \end{bmatrix} = \mathbf{A} \begin{bmatrix} x(k) \\ v_x(k) \\ y(k) \\ v_y(k) \end{bmatrix} + \mathbf{G}\mathbf{w}(k)$$

where,

$$\mathbf{A} = \begin{bmatrix} 1 & dt & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & dt \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and $\mathbf{G} = \mathbf{I}_{4\times4}$. dt is the sampling time. The process noise $w(k)$ has a variance given by

$$\mathbf{Q} = q \begin{bmatrix} dt^3/3 & dt^2/2 & 0 & 0 \\ dt^2/2 & dt & 0 & 0 \\ 0 & 0 & dt^3/3 & dt^2/2 \\ 0 & 0 & dt^2/2 & dt \end{bmatrix}$$

where q is the variance of the zero mean process noise. The above model is a standard discrete time model derived from continuous time target dynamics model. For this model let us assume that the leader node position is given by l_x, l_y. Then extending the state space we have for the joint leader node and the target motion dynamics,

$$\begin{bmatrix} \mathbf{X}(k+1) \\ l_x(k+1) \\ l_y(k+1) \end{bmatrix} = \begin{bmatrix} \mathbf{A} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{X}(k) \\ l_x(k) \\ l_y(k) \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ u_x(k) \\ u_y(k) \end{bmatrix} + \begin{bmatrix} \mathbf{w}_k \\ 0 \\ 0 \end{bmatrix}$$

where $\mathbf{X} = [x, v_x, y, v_y]'$. As in the previous case identify $z_1(k) = x(k) - l_x(k)$ and $z_2(k) = y(k) - l_y(k)$. Then we have,

$$\begin{bmatrix} z_1(k+1) \\ v_x(k+1) \\ z_2(k+1) \\ v_y(k+1) \end{bmatrix} = \mathbf{A} \begin{bmatrix} z_1(k) \\ v_x(k) \\ z_2(k) \\ v_y(k) \end{bmatrix} - \begin{bmatrix} u_x(k) \\ 0 \\ u_y(k) \\ 0 \end{bmatrix} + \mathbf{w}_k$$

Note that $\mathbf{w}_k$ is independent of $z_1(k)$, $z_2(k)$ and $u_x(k), u_y(k)$ and is zero mean.

For the above set-up we wish to minimize the following cost function over N time steps.

$$(Q): \min_{\mathbf{u}_1,..,\mathbf{u}_{N-1}} \mathbf{E}\left(\sum_{k=1}^{N-1} ||\mathbf{u}_k||_1 + ||\mathbf{z}_k||^2 + ||\mathbf{z}_N||^2\right)$$

where $\mathbf{u}_k = [u_x(k), u_y(k)]'$ and $\mathbf{z}_k = [z_1(k), z_2(k)]'$. Let n denote the number of stages left. Then the cost to go at stage $N-n$ for $n=1$ is given by

$$J_n(\mathbf{z}_n, \mathbf{v}_n) = \min_{\mathbf{u}_n} \left\{||u_n||_1 + ||z_n||^2 + ||\mathbf{z}_n + \mathbf{v}_n - \mathbf{u}_n||^2\right\} + \sigma_{\mathbf{w}}^2$$

where $\sigma_{\mathbf{w}}^2 = \mathbf{E}(w(1)+w(2)+w(3)+w(4))^2$; $w(i)$ are the components of the noise vector $\mathbf{w}$. It is easily seen that the optimization problem in the two dimensions separate out. So we will focus on the one dimensional problem instead. For the motion in x direction we have

$$J_n(z_1(n), v_x(n)) - \frac{\sigma_{\mathbf{w}}^2}{2} = \min_{u_x(n)} \left\{|u_x(n)| + (z_1(n))^2 + (z_1(n) + v_x(n) - u_x(n))^2\right\}$$

Similar to the no velocity case, it is clearly seen from the above minimization problem that the optimal control $u_x^*(N-1) = 0$ if $|z_1(N-1) + v_x(N-1)| \leq 0.5$. Outside this region the optimal control is given by,

$$u_x^*(N-1) = z_1(N-1) + v_x(N-1) - \frac{1}{2}\text{sign}(z_1(N-1) + v_x(N-1))$$

Similar policy holds for the optimal control in the y direction. Thus the cost to go function, (for one dimension) at stage $N-1$ is given by

$$J_{N-1}(z_1, v_x) = z_1^2 + \frac{1}{2}\sigma_{\mathbf{w}}^2 + \begin{cases} |z_1 + v_x| - 1/4 & |z_1 + v_x| > 0.5 \\ (z_1 + v_x)^2 & |z_1 + v_x| \leq 0.5 \end{cases}$$

Following the recursion for the cost to go at stage $N-2$ we have

$$J_2(z_1, v_x) = \min_{u_x}[|u_x| + |z_1|^2 + \mathbf{E}_w J_1(z_1 + v_x + w(1) - u_x, v_x + w(2))]$$

From the above minimization it is clear that the cost to go is jointly convex in z_1, v_x. However the symmetry about z_1+v_x is lost. Thus the control at stage $N-1$ is a function of both z_1+v_x and $z_1+2.v_x$. Due to loss of symmetry it becomes difficult to find the nature of the optimal policy for successive stages in this case.

5.1 Policy for a slowly moving target

In the last section we showed that the analysis for random walk with drift is hard to analyze because of loss of symmetry around a single function of the state. Let us have a closer look at the cost to go at stage $N-2$.

$$J_{N-2}(z_1,v_x)=\min_{u_x}[|u_x|+|z_1|^2+\mathbf{E}_w J_1(z_1+v_x+w(1)-u_x,v_x+w(2))]$$

Let $w'=w(1)+w(2)+v_x$. Then we can take the expectation with respect to this noise. Note the now the noise is not zero mean. This will make the cost to go function non-symmetric around $x+v_x$. However if v_x is small then the function is almost symmetric around the mean. In this case we can approximate the cost to go by the following function.

$$\begin{aligned}J_{N-2}(z_1,v_x)=\min_{u_x}\quad & |u_x|+(z_1)^2+\mathbf{E}_{w(1)}(z_1+v_x+w(1)-u_x)^2\\ & +\mathbf{E}_{w'}\begin{cases}|z_1+v_x-u_x+w'|-1/4 & |z_1+v_x-u_x+w'|\leq 0.5\\ (z_1+v_x-u_x+w')^2 & |z_1+v_x-u_x+w'|\geq 0.5\end{cases}\end{aligned}$$

Under the assumption that w' is almost symmetric around zero, the expected cost to go is symmetric around zero. Proceeding by the above approximation to the cost to go function at stage $N-2$ is given by,

$$J_{N-2}(z_1,v_x)=\sigma^2_{w(1)}+(z_1)^2+\begin{cases}|z_1+v_x|-\Gamma_2+\mathbf{E}_{w(1),w(2)}J_{N-1}(\Gamma_2+w) & |z_1+v_x|\leq\Gamma_2\\ \mathbf{E}_{w(1),w(2)}J_{N-1}(z_1+v_x+w) & |z_1+v_x|\geq\Gamma_2\end{cases}$$

We do not switch if $|z_1+v_x|\leq\Gamma_2$ for some $\Gamma_2>0$.

Remark 5.1. If the velocity is not small then we get asymmetric switching regions, i.e. do not switch if $\Gamma_- \leq (z_1 + v_x) \leq \Gamma_+$. The values Γ_- and Γ_+ depend on the sign of the velocity (as well as the magnitude of it) as this sign is responsible for the negative or positive bias in the noise. In particular if the velocity is positive then the bias is positive and Γ_+ decreases and vice versa.

Recursively at any stage $N - k$ we have,

$$J_{N-k}(z_1, v_x) = \min_{u_x} \left\{ |u_x| + (z_1)^2 + \mathbf{E}_{w(1),w(2)} J_{N-k+1}(z_1 + w(1) + v_x, v_x + w(2)) \right\}$$

First note the following points. If J_{N-k+1} is jointly convex in $z1$ and v_x then J_{N-k} is jointly convex in $z1$ and v_x. Assuming that v_x is small, the cost to go J_{N-k+1} becomes approximately a function of $z_1 + v_x$. Note that J_{N-k+1} is symmetric with respect to replacing z_1 by $-z_1$ and v_x by $-v_x$. Under the assumption of v_x small, $\mathbf{E}_{w(1),w(2)} J_{N-k+1}(z_1 + w(1) + v_x, w(2))$ is almost symmetric around $z_1 + v_x = 0$. Thus there exists a threshold Γ_k such that for $|z_1 + v_x| \leq \Gamma_k$, $u_k^* = 0$. It is not obvious in this case that $\Gamma_k > 0$ for all k.

6 Incorporating the error covariance in the analysis

The nature of the optimization problem gives us a handle on incorporating the error covariance into the analysis. To see this observe that under perfect observation of state z for random walk,

$$J_n(z) = z^2 + \min_u \{|u| + \mathbf{E}_w J_{n-1}(z + w - u)\}$$

Using the CEC policy we are essentially running a recursive algorithm to estimate the mean target position and error covariances. At step n one can incorporate the uncertainty in the estimated state by adding an extra noise term w_n of variance equal to the current error covariance in the cost to go, i.e.,

$$J_n(z) = z^2 + \min_u \{|u| + \mathbf{E}_{w+w_n} J_{n-1}(z + w_n + w - u)\}$$

The effect of the additional noise term in the expectation leads to more smoothing of the cost to go function. In the ϵ neighborhood of z, the derivative is thus reduced. This leads to expansion of the switching region in general.

In the next section we will show the simulation results for the two cases, viz., (1) Random walk, (2) Random Walk with drift. For simulations in the case of random walk with drift we apply a one step receding horizon control via certainty equivalence.

7 Simulations

7.1 Random Walk without drift

The simulation set up is as follows. The sensor network is a sensor grid consisting of 81 sensors placed on integer co-ordinates in $[-3, 3] \times [-3, 3]$. For the random walk, uniform noise between [-0.25,25] was chosen as the process noise. The initial state was at [0,0] with initial covariance of 10 in each direction. The observation model at each sensor is given by,

$$y(x_p, y_p) = \frac{20}{1 + \sqrt{2}(((X - x_p)^2 + (Y - y_p)^2))} + v$$

where v is a gaussian noise of variance 1 in each dimension and x_p and y_p are the x and y co-ordinates of the sensor. The switching threshold was kept constant to 0.2 over the 50 time steps. We run two algorithms.

1. At each time instant 4 nodes around the leader nodes report their observations. We use an Extended Kalman Filter (EKF) to update/predict the state. We employ certainty equivalence control where the control action for switching uses the state estimate given by the EKF. The leader node is switched to the predicted location if the distance between the current leader node and the current predicted position given by the EKF exceeds 0.2.

2. We compare the performance of the above algorithm to a hindsight policy where each time 4 sensors around the true target position were selected to report to the leader node. The leader node then uses an EKF for estimating the target position. This policy reflects the optimal state estimate with 4 sensors since the optimal sensor locations are provided as side information. However, it does not account for the communication cost.

The communication cost (in bit-meters) for switching is given by the total distance traveled by the leader node in 50 time steps. The results are shown in figure 3 where the statistics are averaged over 50 Monte Carlo runs. As seen the tracking error for both schemes is similar. However the communication cost for hindsight EKF policy turns out to be 12.3 bit-meters while communication cost for our deadzone adaptive switch is 4.2 bit-meters, thus providing a significant energy savings even when compared with hindsight EKF.

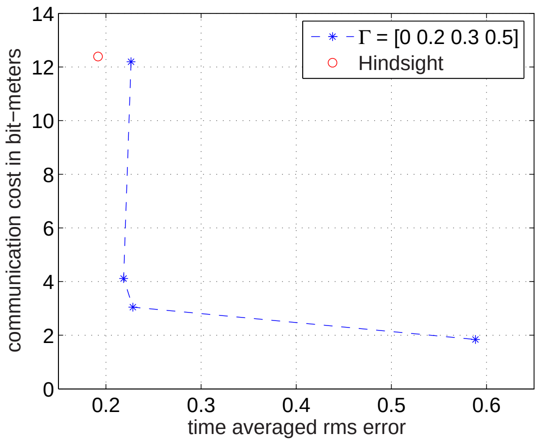


Figure 3: The tradeoff between the communication costs and the error performance for the random walk (no drift) case.

7.2 Random Walk with Drift

The simulation set-up is as follows. The sensor network consists of a two dimensional grid in $[-45, 45] \times [-45, 45]$ meter square area, where the sensors are placed at a distance of $\rho = 3$ meters apart. The measurement model is given by

$$y(x_p, y_p) = \frac{20}{1 + (((X - x_p)^2 + (Y - y_p)^2))/(R_s^2)} + v$$

where v is a gaussian noise of variance 1 in each dimension and x_p and y_p are the x and y co-ordinates of the sensor. X, Y are the co-ordinates of the target location. For simulations $R_s^2 = 10$. This ensures that each sensor has a sensing range a little beyond the next sensor. This also ensures that one can vary the threshold Γ to a certain point without loosing the observability in the system. For the target dynamics $q = 0.01$ and $dt = 1$. The target starts at $(0, 0)$ with a velocity of 0.25 in both x and y directions.

Each time 6 sensors around the leader node were selected to report their observations in each of the cases, viz., (1) hindsight policy and (2) adaptive switching employing the one step receding horizon control via certainty equivalence. As shown from figure 4 there are around 10 percent gains over the hindsight policy in the communication costs without suffering much in the error performance. Each point on the plot is an average over 27 Monte Carlo runs. It is important to note that for the actual tracker the gain by using a threshold policy ($\Gamma = 0.5$) over a always switch policy ($\Gamma = 0$) is 20 percent for similar error performance.

For the simulations if $\Gamma > 3.5$ then the rms errors blew up along with the communication costs. This is because the observability of the state is lost due to limited sensing range of the sensors.

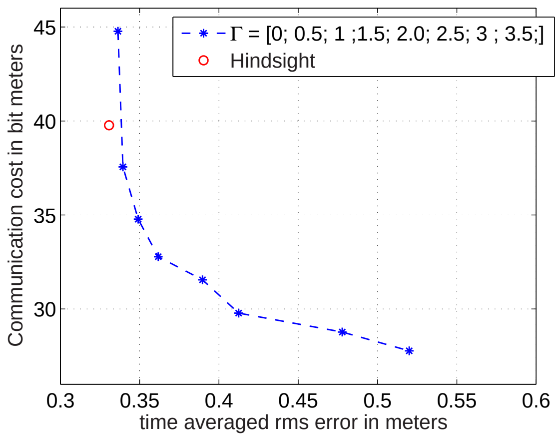


Figure 4: The tradeoff between the communication costs and the error performance for the random walk with drift case. Applying the one step lookahead policy on the estimated state.

8 Conclusions

In this paper we abstracted a tractable distributed target tracking problem that retains essential features of the original problem, i.e. the tradeoff between estimation error and the cost of communication. It was shown that under simple model of random walk for target motion that the optimal (in the certainty equivalence sense) policy consists of a "footprint" region around each sensor such that the sensor holds and processes the observations unless the target moves outside the footprint.

This leads to considerable energy savings which are reported via simulations. From the figures 3 and 4 one can see that there is a "knee" in the communication cost (bit-meters) vs rms error curve. ¿From a designer's perspective this is the point at which one can operate the network for target tracking problems.

Future work includes finding the complete solution to the case when the target motion has a drift term. Our conjecture in this case is that the optimal policy consists of a skewed footprint around each sensor that depends on the direction and magnitude of the velocity.

9 Appendix

9.1 Proof of theorem 4.1

Let n denote the number of stages left. Then we have

$$J_n(z) = \min_u \left\{ \lambda |u| + z^2 + \mathbf{E}(J_{n-1}(z + w - u)) \right\} \tag{5}$$

The following properties hold for the n-stage cost-to-go function:

Lemma 9.1. *The n-stage cost-to-go function, $J_n(\cdot)$ is real valued, positive, convex, & symmetric about $z = 0$ and monotonic for $z \geq 0$.*

Proof. That $J_n(\cdot)$ is real-valued for all $z \in \mathbb{R}$ is straightforward (just substitute $u = 0$ as the policy for all stages upto n). The rest of the proof follows through induction. It is easy to see that these properties hold for $n = 1$. Suppose, these properties hold for all stages $k < n$. Then the cost-to-go for $k = n$ is convex. This follows from the fact [6] that minimizing a jointly convex function in two variables results in a convex function in one variable, i.e., $f^*(x) = \min_y f(x, y)$ is a convex function of x. Symmetry follows from the fact that the noise w has a symmetric distribution and $J_{n-1}(\cdot)$ is a symmetric function by hypothesis and z^2, $|u|$ are symmetric functions. This is because, $\mathbf{E}J_{n-1}(z-u+w) = \mathbf{E}J_{n-1}(-(z-u)+w)$. Therefore, if $u = u^*$ is a minimizer for the case $z = z_0 > 0$ and $J_n(z_0)$ the corresponding cost then, $u = -u^*$ matches this cost for $z = -z_0 < 0$ and vice versa.

To establish monotonicity we observe that a positive, symmetric, convex function about zero is monotonically increasing for $z > 0$. To see this, consider, $0 < z_1 < z_2$, and $\theta = 0.5(1 - x/y)$,

$$J_n(z_1) = J_n(\theta(-z_2) + (1-\theta)(z_2)) \leq \theta J_n(-z_2) + (1-\theta)J_n(z_2) = J_n(z_2)$$

Strict monotonicity follows from the fact that z^2 is strictly monotonic. □

Lemma 9.2. *The n-stage cost-to-go, $J_n(\cdot)$ is differentiable almost everywhere (with respect to lebesgue measure) and*

$$\frac{\partial}{\partial \eta}\mathbf{E}J_n(\eta + w) = \mathbf{E}\frac{\partial}{\partial \eta}J_n(\eta + w) = \frac{1}{2\alpha}(J(\eta+\alpha) - J(\eta-\alpha))$$

Proof. That $J_n(\cdot)$ is differentiable is Proposition 17 of [**?**] and follows from the fact that left and right derivatives always exist for convex functions and they match except on a set of lebesgue measure zero.

The second fact is a consequence of dominated convergence theorem. To see this define:

$$f_{1/k}(z) = \frac{J_{n-1}(z + w_n + 1/k) - J_{n-1}(z + w_n)}{1/k},\ k \in Z^+,\ z > 0$$

Now, $f_{1/2}$ is positive, integrable in any finite interval and dominates $f_{1/k}$ for $k \geq 2$. Furthermore,

$$f_{1/2} \geq f_{1/3} \geq \cdots f_{1/k} \geq \cdots \geq 0$$

and therefore, $f_{1/k}(\cdot)$ is pointwise convergent. Therefore, all conditions for dominated convergence are satisfied. The final result is obtained by substituting the parameters for uniform random noise and computing the integral. □

Lemma 9.3. *The n-stage optimal policy is a switching policy, i.e.,*

$$\begin{aligned} u &= 0,\ |z| \leq \Gamma_n \\ u &= (|z| - \Gamma_n)sign(z),\ |z| \geq \Gamma_n \end{aligned}$$

where, $\Gamma_n > 0$*. The corresponding n-stage cost-to-go is described by:*

$$J_n(z) = \begin{cases} z^2 + \mathbf{E}(J_{n-1}(z+w)) & |z| \le \Gamma_n \\ \lambda|z| - \Gamma_n + z^2 + \mathbf{E}J_{n-1}(\Gamma_n + w) & |z| > \Gamma_n \end{cases}$$

Proof. The necessary and sufficient condition for optimality is that the sub-gradient set contain the element zero(see page 264 in [6]). The optimality of switching policy follows from the fact that zero is an interior point of the sub-gradient of the argument in the minimization problem of Equation 5. To show this let,

$$\mathcal{L}(u) = \left\{\lambda|u| + z^2 + \mathbf{E}(J_{n-1}(z+w-u))\right\}$$

We consider the sub-gradient set for sufficiently small $|z|$ and $u = 0$,

$$\partial\mathcal{L}(0) = [-\lambda, \lambda] + \left\{\frac{\partial}{\partial u}\mathbf{E}(J_{n-1}(z+w-u))\right\}_{u=0} \stackrel{(a)}{=} [-\lambda, \lambda] + \frac{1}{2\alpha}(J_n(z+\alpha) - J_n(z-\alpha)) \quad (6)$$
$$\stackrel{(b)}{=} [-\lambda, \lambda] + \frac{1}{2\alpha}(J_n(\alpha+z) - J_n(\alpha-z))$$

where (a) follows from Lemma 9.2 and (b) follow from symmetry property (Lemma 9.1). Now we know that $J_n(\cdot)$ is continuous (since convex real-valued functions are continuous). Therefore, for sufficiently small z, the last term in the final expression can be made arbitrarily small. Consequently, in this region the cost-to-go is given by:

$$J_n(z) = z^2 + \mathbf{E}(J_{n-1}(z+w)), \ \forall \ |z| \le \Gamma_n$$

For $z \ge \Gamma_n \ge 0$ we again compute the sub-gradient and determine conditions for sub-gradient set to contain zero:

$$\partial\mathcal{L}(u) = 0 \implies \lambda - \frac{1}{2\alpha}(J_n(\alpha+z-u) - J_n(z-u-\alpha)) = 0, \ \forall \ z \ge \Gamma_n$$

This implies that, $z - u = \Gamma_n$. The cost-to-go function follows by direct substitution. □

To complete the proof we need to establish that Γ_n is bounded away from zero independent of n. Let $\Gamma_* = \inf_n \Gamma_n$.

Lemma 9.4.

$$\Gamma_* \geq \Gamma_0 = \min\left(\frac{\alpha}{2}, \frac{\alpha\lambda}{\alpha+\lambda}\right)$$

Proof. Assume there is an n such that, $\Gamma_n \leq \alpha/2$. If not then the proof is established. Consider, for $0 \leq z \leq \Gamma_0$

$$\frac{1}{2\alpha}(J_n(\alpha+z) - J_n(\alpha-z)) \overset{(a)}{=} \frac{1}{2\alpha}\left(\lambda|\alpha+z| + (\alpha+z)^2 - \lambda|\alpha-z| - (\alpha-z)^2\right) = z\frac{\alpha+\lambda}{\alpha\lambda} \leq \lambda,\ \forall\, z \leq \Gamma_0$$

where, (a) follows from Lemma 9.3 and noting the fact that $\alpha - \Gamma_n \geq \Gamma_n$. Therefore, whenever $z \leq \Gamma_0$ the sub-gradient set always contains the zero element for the zero policy, i.e., $u = 0$. This implies that $\Gamma_n \geq \Gamma_0$. □

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