Reliable Tracking with Intermittent Communications

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***Abstract*— We consider the problem of distributed target tracking of a linear dynamical system via networked sensors. Our setup consists of a set of sensors connected to a fusion center by means of communication links. Unreliable communication channels leads to communication delays and loss of information. To address this problem we model the arrival of messages from the sensors to the fusion center by a random process. The question arises as to what messages to encode for the fusion center. One possibility that has recently received much attention is to transmit local observations to the fusion center, which then fuses these intermittent observations through Kalman Filtering techniques. In contrast we develop a scheme for fusion of intermittent local state estimates. The salient aspects of scheme are: (a) Efficiency, i.e., covariance of local estimate is no larger than covariance of observation; (b) Robustness, i.e., the error covariance is bounded with high probability even under vanishing link-connectivity. In contrast fusion of intermittent observations leads to unbounded errors even for moderate values for link-connectivity; (c) Scalable performance, i.e., a K-node sensor network with K guaranteed communication links is inferior to an N-node sensor network with K functioning communication links on average.**

I. Introduction

We consider a collection of sensors that collectively track a linear system that is driven by noise. The measurement model is also linear, hence the optimal MMSE centralized solution is given by the canonical Kalman filter. In this paper we consider distributed tracking with a collection of sensors connected to a fusion center by means of communication links. The communication links are unreliable and messages sent to the fusion center from the sensors are subject to packet losses, with no retransmission. The question arises as to what messages to encode for the fusion center at each sensor.

The general problem has received significant attention (see [3], [5], [7], [8], [1], [9], [10] and references therein). Of particular relevance to our scenario is the recent work on Kalman Filtering with intermittent observations [9]. In their setup the sensors directly send their local observations to the fusion center but the arrival of these messages are subject to random packet losses. The fusion center fuses these intermittent observations through Kalman Filtering techniques. They study the statistical convergence properties of the estimation error covariance, showing the existence of a critical value for the arrival rate of the observations, beyond which a transition to an unbounded error occurs. Moreover, beyond this critical regime the likelihood that the error remains bounded goes to zero as well.

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We consider the setup where each sensor transmits local sufficient statistics (i.e. summary of past local observations) rather than the raw measurements. The practicality of this scheme is motivated by recent trends in sensing, computing and communication technology, where it is becoming increasingly clear that communication-energy/bit far outstrips computational as well as sensing energies. Our scheme has several advantages over transmission of raw measurements. First, it is efficient, i.e., covariance of local sufficient statistics is no larger than covariance of observation. Second, since the fusion center receives a summary of local measurements rather than the instantaneous observation, the performance of our scheme is superior. Third, the scheme is robust to communication link failures and the error covariance is bounded with high under vanishing link connectivity. In contrast fusion of intermittent observations leads to unbounded errors even for moderate values for link connectivity. Finally, our scheme leads to scalable performance, i.e., we show that an N-node sensor network with $K < N$ random operational links to the fusion center at any instant of time has significantly higher performance than K-node sensor network with guaranteed connectivity.

The main difficulty of our proposed scheme is in optimally fusing the intermittent local sufficient statistics received at the fusion center. These difficulties arise due to: (a) Conditional independence: It is well known in gaussian estimation problems that if the local measurements are conditionally independent when conditioned on the underlying parameter, the optimal fusion rule is a linearly weighted average of the optimal local estimates. However, when the underlying state (or parameter) is random and time varying the local measurements are no longer conditionally independent when conditioned on the current state. We deal with this scenario by appealing to ideas developed in [3], [5]. In particular, we show that conditional independence can be recovered if the conditioning variable includes the previous state estimate. (b) Random Arrivals: This leads to messages that arrive at the fusion center from random collections of sensors. Due to message losses, the fusion center will not have local estimates from each sensor corresponding to a common time; the local state estimates are no longer synchronized and the fusion rule has to be substantially modified. We present two different schemes to deal with this issue. In the first scheme, a compensation is transmitted by each sensor in addition to local state estimate. The second scheme is a suboptimal scheme wherein local state estimates are fused by incorporating their instantaneous cross-correlation.

II. Problem statement

Let $\mathbf{R}^n$ denote n-dimensional real vectors, and denote by $\mathbf{M}^{n\times n}$ the set of symmetric positive-definite matrices of dimension $n \times n$. We consider the discrete-time system

$$X_{t+1} = AX_t + W_t, \quad X_0 \sim N(0, \Sigma_0), \tag{1}$$

where A is an $n \times n$ matrix and $W = (W_t : t = 0, 1, 2, \cdots)$ is an IID sequence such that $W_t \sim N(0, \Sigma_W)$, independently of X_0.

We consider tracking the sequence $(X_t : t = 0, 1, 2, \cdots)$ based on measurements taken by a collection V of sensors. The measurement of sensor $v \in V$ taken at time slot t is denoted by $Y_t(v) \in \mathbf{R}^m$ and it satisfies

$$Y_t(v) = C_t(v)X_t + U_t(v), \quad v \in V,$$

where $C_t(v)$ is an $m\times n$ matrix, and $(U_t(v) : t = 0, 1, 2, \cdots)$ is an IID sequence such that $U_t(v) \sim N(0, \Sigma_U)$. In particular if all measurements are immediately available to a central processor then the MMSE estimator is a Kalman filter. Specifically, the MMSE estimate $X_{t|t} = E[X_t|Y_\tau(v) : v \in V, \tau \leq t]$ of X_t based on $Y_\tau(v),\ v \in V, \tau \leq t$, satisfies

$$X_{t|t} = X_{t|t-1} + P_{t|t} \sum_{v\in V} C_t^T(v)\Sigma_U^{-1}(Y_t(v) - C(v)X_{t|t-1}) \tag{2}$$

where $P_{t|t} = E(X_t - X_{t|t})(X_t - X_{t|t})^T$ is the conditional error covariance matrix at time t and is given by the recursion:

$$P_{t|t}^{-1} = P_{t|t-1}^{-1} + \sum_{v\in V} C_t^T(v)\Sigma_U^{-1}C_t(v)$$

These steps are commonly referred to as the measurement update steps. To complete the Kalman filter updates we require the so called prediction steps, which are given by:

$$X_{t+1|t} = AX_{t|t},\ \ P_{t+1|t} = AP_{t|t}A^T + \Sigma_W \tag{3}$$

To complete the problem setup we need to describe the communication connectivity model. The connections at every instant of time is an IID process. In one model we assume that the connection between each sensor, v, and the fusion center are independent Bernoulli processes, $\{\gamma_k(v)\}$, $k = 0, 1, \ldots,$ with parameter p. We also consider an alternative bulk-model where the number of connections, K, at any instant of time is fixed and the set of K sensors that connect to the fusion center, $\mathcal{C}_K = \{v_1, v_2, \ldots, v_K\} \subset V$ are chosen independently at each instant of time. For convenience we define the indicator function $I_v(t)$ to denote whether or not the sensor, v, is operational at time t and $N_v(t) = \max\{k \in [0, t] \mid I_v(k) = 1\}$ to denote the final arrival time of vth sensor before time t. Let $\mathcal{Y}_v^k$ denote the sensor v's measurement upto time k and $\mathbf{N}(t) = (N_1(t), N_2(t), \ldots N_|V|(t))$ the vector of final arrival times. Our main objective is to compute the optimal estimate,

$$X_{t|\mathbf{N}(t)} = E(X(t) \mid \mathcal{Y}_v^{N_v(t)}, \forall\, v \in V)$$

By convention if no arrival occurs before t the conditioning event $\mathcal{Y}_v^{N_v(t)}$ is omitted.

Optimal Performance: The above problem formalizes the notion of optimal performance in that it characterizes recovery of centralized performance through decentralized processing. To explain this point further, note that in the bernoulli model with $|V|$ sensors the average number of messages received by the fusion center in T units of time is $p|V|T$. Nevertheless, since each sensor encodes the history of its local observations the fusion center potentially has access to $|V|T$ observations irrespective of the size of connection probability p for sufficiently large T. These gains are realized when there exist schemes whereby fusing the local estimates/messages (which are summaries of locally observed data) is equivalent to centrally processing the corresponding raw observations, i.e., all past observations $\tau \leq N_v(t),\ \forall\ v\ \in V$. The following section presents several situations where the optimal centralized estimate can be exactly recovered through distributed processing.

III. Communication Protocols and Fusion Algorithms

In this section we outline several cases and present protocols for which the centralized performance is achievable. The fundamental issues are: (A) Lack of conditional independence of local observations at different sensors conditioned on the current state; (B) Fusion of unsynchronized messages.

No Process Noise & Parameter Estimation Problems: This is the case where $W(t) = 0$ in Equation 1, i.e., $X(t + 1) = AX(t)$. The parameter estimation problem corresponds to setting $A = I_n$. The main feature here is that the time history of past observations, i.e., $\mathcal{Y}^{N_v(t)}$, $v \in V$ are conditionally independent given current state. Therefore, the optimal protocol in this case is to send the local estimates and the fusion rule is given by:

$$X_{t|\mathbf{N}(t)} = P_{t|\mathbf{N}(t)} \sum_{v\in V} (P_{t|N_v(t)}^v)^{-1} X_{t|N_v(t)}^v$$

where, $P_{t|\mathbf{N}(t)}, P_{t|N_v(t)}^v$ are the global and local covariance error matrices based on data upto $\mathbf{N}(t)$ and $N_v(t)$ respectively. It is interesting to compare different possibilities to highlight the fundamental differences. We consider two scenarios: (a) a single sensor with guaranteed connection to a fusion center, (b) $|V|$ sensors with unreliable links with bernoulli parameter, $p = 1/|V|$. For Case (b) the average number of samples in time T is $p|V|T = T$. For the latter scenario consider two possibilities: transmitting raw observations vs. transmitting local MMSE estimates.

Theorem III.1 *Suppose the average MMSE errors (where the average is taken w.r.t. number of samples) in these three cases are denoted as $MMSE_{single}$, $MMSE_{raw}$, $MMSE_{local}$. Then:*

$$E(P_{t|\mathbf{N}(t)}) = MMSE_{local} = \frac{1}{|V|} MMSE_{raw} \overset{t\to\infty}{\longrightarrow} \frac{1}{|V|} MMSE_{single}$$

Singe Remote Sensor: Here we consider a single remote sensor, l, transmitting messages to a fusion center, which also has a local sensor, f. This case is interesting because the observations at the two sensors are no longer conditionally independent. Therefore, the messages have to be compensated to account for the dependence. The main idea is based on the information form of the Kalman filter, which we briefly explain. The optimal global and local Kalman filter update equations satisfy:

$$(P_{t|t})^{-1}X_{t|t} = (P_{t|t-1})^{-1}X_{t|t-1} + C_l^T\Sigma_U^{-1}y_l + C_f\Sigma_U^{-1}y_f$$
$$(P_{t|t}^l)^{-1}X_{t|t}^l = (P_{t|t-1}^l)^{-1}X_{t|t-1}^l + C_l^T\Sigma_U^{-1}y_l$$
$$(P_{t|t}^f)^{-1}X_{t|t}^f = (P_{t|t-1}^f)^{-1}X_{t|t-1}^f + C_f^T\Sigma_U^{-1}y_f$$

where, $(P_{t|t})^{-1} = (P_{t|t-1})^{-1} + C_l^TR^{-1}C_l + C_f^TR^{-1}C_f$. It follows from straightforward algebraic modification that the fused sensor update can be re-written in terms of local estimates.

$$X_{t|t} = X_{t|t-1} + P_{t|t}\sum_{v\in\{l,f\}}(P_{t|t}^v)^{-1}(X_{t|t}^v - X_{t|t-1}^v)$$
$$- P_{t|t}\sum_{v\in\{l,f\}}C_v^T\Sigma_U^{-1}C_v(X_{t|t-1} - X_{t|t-1}^v)$$

This would be the optimal fusion algorithm for the no packet loss case. Thus the messaging protocol will be identical to the previous situation but the fusion algorithm will be dynamic as it uses previous communicated estimates. We now describe the scheme to compensate for packet losses. Notice that under packet losses the local state estimates at the previous time is not generally available. This requires compensation of the message as well as the fusion algorithm. It turns out that the fusion algorithm satisfies:

$$(P_{t|\mathbf{N}(t)})^{-1}X_{t|\mathbf{N}(t)} = (P_{t|t}^f)^{-1}X_{t|t}^f + (P_{t|\mathbf{N}(t)})^{-1}S_f(t)$$
$$+ \underbrace{(P_{t|N_l(t)}^l)^{-1}X_{t|N_l(t)}^l + (P_{t|\mathbf{N}(t)})^{-1}A^{t-N_l(t)}S_l(N_l(t))}_{\text{Remote Sensor Message}} \quad (4)$$

Note, that $N_f(t) = t$ for the sensor at the fusion center. The compensation term $S_l(t)$ is computed locally by the local sensor as follows:

$$S_l(t) = (I - P_{t|\mathbf{N}(t)}C_v\Sigma_U^{-1}C_v)AS_k(v) + L_tX_{t|t}^v \quad (5)$$

where, L_t is a matrix that depends on the system dynamics as well as the error-covariance matrices. Note that in the fusion rule the compensation and state estimate for the remote sensor is predicted based on last message received.

Multiple Remote Sensors: The main difficulty in generalizing the single remote sensor case to multiple sensors is that this at the very least requires knowledge of the final arrival times $\mathbf{N}(t)$. One can observe this aspect from Equation 5 for the compensation. This involves the covariance term $P_{t|\mathbf{N}(t)}$. For the bulk model with two sensors where only one sensor communicates information at any instant of time this is not an issue. This is because whenever a sensor is unsuccessful, this implies that the other sensor is successful. Therefore $\mathbf{N}(t)$ is revealed by the system without the need for a feedback mechanism. There are other situations where $\mathbf{N}(t)$ is partially known such as Carrier Sense Multiple Access protocols are used for communication. In this situation although each sensor senses the transmit times, it does not know whether these transmit times correspond to different sensors. In this situation one may modify the existing protocols so that each sensor after a transmission ceases transmission until it has sensed a given number of transmissions. This ensures diversity in sensing information. With any such modification wherein $\mathbf{N}(t)$ can be either sensed or fedback to the sensor from the fusion center it turns out that the optimal messaging for each sensor is given by Equation 5. The fusion rule in this case is similar to Equation 4:

$$(P_{t|\mathbf{N}(t)})^{-1}X_{t|\mathbf{N}(t)} = \sum_{v\in V}(P_{t|N_v(t)})^{-1}X_{t|N_v(t)}^v$$
$$+ \sum_{v\in V}(P_{t|\mathbf{N}(t)})^{-1}A^{t-N_v(t)}S_v(N_v(t))$$

Our main result is summarized in the following theorem:

Theorem III.2 *Consider the fusion algorithm described in the previous section and suppose the packet losses follow the bernoulli model described in Section 2. Furthermore, let the N sensors be identical and (A, C) form a detectable pair. We then have, for all values of link loss probability that,*

$$\mathit{Prob}\{\liminf P_{t|\mathbf{N}(t)} = P_\infty\} = 1$$

where, P_∞ is the asymptotic covariance when all the sensor measurements are centrally available. If the packet loss probability, $p^N\lambda_{max}(A) > 1$ then, $E(P_{t|\mathbf{N}(t)}) \longrightarrow \infty$.

The above result implies that irrespective of the packet loss probability there exists time instants when the error covariance of the decentralized scheme is arbitrarily close to the centralized scheme. These time instants can be thought of as renewal periods and the expected length of the renewal period depends only on the loss probability and not on the underlying dynamics. In contrast if local raw measurements are transmitted and are subject to random losses, the error covariance is always arbitrarily large. The second result states that the mean value of the error covariance is unbounded if the packet loss probability is small relative to the system dynamics.

Fig. 1. Example of transition to instability for a single sensor scenario with packet losses for different dynamical models.

IV. Simulation Results

A simulation of a one-sensor scheme is provided in Figure 1. Here we have two choices for $A = -1.25, 1$ corresponding to an unstable system dynamics and a random walk model. Also, $C = 1$, $R = 2.5$ and $Q = 1$. The critical value for the transition for the unstable system is a link loss probability of $p = 0.64$ and for the random walk it is $p = 1$.

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