Frame Synchronization for Noncoherent Demodulation on Flat Fading Channels

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Abstract – In this work we consider the problem of locating a known periodically embedded synchronization pattern in random data, which is subject to flat fading and additive white Gaussian noise. We derive the maximum likelihood rule and its high-SNR approximation for the case of noncoherent demodulation and show experimental results for the slow-fading Rayleigh channel. It is shown that the well-known correlation rule performs substantially worse than maximum-likelihood rule derived here.

I. INTRODUCTION

The problem of locating synchronization patterns is important in such areas as wireless communications, and has been studied in detail. An early work by Barker [1] considered the problem of finding a known synchronization pattern in random data and proposed a correlation-type pattern recognizer. In [2] Massey derived the maximum-likelihood (ML) rule for detecting periodically embedded synchronization patterns for the AWGN channel with coherent BPSK modulation. He has shown that when ML rule is used, synchronization pattern detection performance is substantially better than that of the previously proposed correlation rule, earlier believed to be optimal. Shortly thereafter, Norden [3] analyzed Massey’s results and obtained the performance limits of both the ML and the correlation rules. He also found that an ML rule approximation at high SNRs derived by Massey provides essentially the same performance as the optimal rule but is much simpler to implement. Later, Lui and Tan [4] extended Massey’s derivation to include M-ary digital communication systems and derived the ML rule for both coherent and noncoherent phase demodulation. Robertson [5] derived the ML rule for the Rayleigh fading channel when M-ary coherent phase signaling is used. In addition to deriving the ML rules, the authors in [4, 5] derived high-SNR approximations to the optimal ML rules that performed nearly as well as the optimal rules. Recently, Matzner, Eck, and Changsong [6] have considered the problem of locating aperiodically embedded synchronization patterns.

In this paper we extend the results of these earlier works and present the derivation of the ML rule for a flat Rayleigh fading channel when noncoherent demodulation is employed. It is assumed that perfect channel state information is available. Since exact analysis of the performance of the synchronization rules does not appear to be feasible [4], we present results of computer simulations to evaluate performance of the derived ML rule. In the simulations we employed the synchronization pattern used in [4] and, additionally, examined the results of detecting the MPEG-4 start code prefix [7]. The motivation behind this is to provide a basis for future work in joint source-channel decoding.

II. DERIVATION OF THE MAXIMUM LIKELIHOOD RULE

We consider a channel consisting of multiplicative flat fading followed by additive white Gaussian noise. Data is transmitted using an M-ary orthogonal modulation scheme in frames of fixed size N, with the first L symbols being the known synchronization pattern and the remaining N−L symbols being the random data. As in [4], we make use of the fact that for Gaussian channels each M-ary symbol can be represented by a K-dimensional vector, with K ≤ M. We denote our set of signal vectors \{W_j | 1 ≤ j ≤ M\}. Using this notation, each frame is of form \[S_0S_1 \cdots S_{L-1}d_L \cdots d_{N-1}\], where L symbols, \(S_0 \cdots S_{L-1}\), form the frame synchronization pattern and the remaining \(N−L\) symbols, \(d_L \cdots d_{N-1}\), are the random data. In order to simplify analysis, we assume that all data symbols are equally likely. Typically, no restriction is made on the possibility of synchronization patterns appearing in the random data portion of the frame [2–5]. It is well-known, however, that in practical implementations such as MPEG-4 coding which is considered here, the synchronization pattern is not allowed to reappear in the random data. Our simulations took this fact into account by ensuring that no synchronization pattern appears in the random data portion of the frame.

For the noncoherent demodulator the output sequence corresponding to an arbitrarily selected portion of \(N\) transmitted signal vectors can be represented by the
vector \( \mathbf{r} = (r_0 r_1 \cdots r_{N-1}) \), where \( r_i = (r_{iL}, r_{iQ}) \) for \( 0 \leq i \leq N-1 \). This output sequence is composed of the corresponding sequence of transmitted vectors, each multiplied by a complex-valued fading value \( \tilde{a}_i, 0 \leq i \leq N-1 \), and added to a sequence of mutually independent random noise vectors \( \mathbf{n}_i = (n_{iL}, n_{iQ}) \). We assume independent channel states for each symbol. This can be achieved, for example, by using a perfect interleaver. To achieve similar results Robertson [5] assumed that the synchronization pattern is spread throughout the frame.

The in-phase and quadrature components of the demodulator output corresponding to the transmitted signal vector \( \mathbf{d}_i, 0 \leq i \leq N-1 \) are \( r_{iL} = a_i d_i \cos \theta_i \) + \( n_{iL} \) and \( r_{iQ} = a_i d_i \sin \theta_i + n_{iQ} \), respectively. We assume that the components of the noise vectors are i.i.d. with zero mean and variance \( N_0/2 \), where \( N_0 \) is the one-sided power spectral density. The signal carrier phases \( \theta_i, 0 \leq i \leq N-1 \) are independently uniformly distributed random variables on \([0, 2\pi])\). The receiver observes any \( N \) consecutive channel outputs and determines the most likely beginning of a new frame which must occur somewhere in \( \mathbf{r} \). To account for periodic frame boundaries, all operations on indices will be modulo \( N \) [2]. Our frame synchronizer will generate an estimate, \( \hat{\mu} \), in \( \{0, 1, \ldots, N-1 \} \) that maximizes the conditional probability of receiving the vector \( \mathbf{r} \) given that \( \mu \) was the start of a frame, i.e., \( p(\mathbf{r}|\mu) \). This can be expressed by the maximum-likelihood (ML) rule similar to that given in [2, 4]

\[
p(\mathbf{r}|\mu) \approx \sum_{\mathbf{d} \in \mathcal{M}} p(\mathbf{r}|\mu, \mathbf{d}) M^{-(N-L)},
\]

where \( \mathcal{M} \) is the signal alphabet size and the approximation is due to the fact that we do not allow the resynchronization pattern to reappear in the random data portion of the frame. The approximation is a very close one since typically \( N \gg L \).

Because noise is assumed to be Gaussian, \( p(\mathbf{r}|\mu, \mathbf{d}, \mathbf{\theta}) \) can be expressed as

\[
p(\mathbf{r}|\mu, \mathbf{d}, \mathbf{\theta}) = \frac{1}{(\pi N_0)^N} \prod_{i=0}^{L-1} \exp \left\{ -\frac{1}{N_0} \left[ \left( \| \mathbf{r}_{iL} \mathbf{d} - \mathbf{S}_i \mathbf{a}_{iL} \cos \theta_i \| \right)^2 \ight. \\
\left. + \left( \| \mathbf{r}_{iQ} - \mathbf{S}_i \mathbf{a}_{iQ} \sin \theta_i \| \right)^2 \right] \right\} \prod_{i=L}^{N-1} \exp \left\{ -\frac{1}{N_0} \left[ \left( \| \mathbf{r}_{iL} \mathbf{d} - \mathbf{S}_i \mathbf{a}_{iL} \cos \theta_i \| \right)^2 \ight. \\
\left. + \left( \| \mathbf{r}_{iQ} - \mathbf{S}_i \mathbf{a}_{iQ} \sin \theta_i \| \right)^2 \right] \right\},
\]

where \( \| \cdot \| \) is the Euclidean norm and \( a_{iL} \) is the magnitude of \( \tilde{a}_{iL} \). Averaging with respect to \( \mathbf{\theta} \) and simplifying and rearranging the resulting expression yields

\[
p(\mathbf{r}|\mu, \mathbf{d}) = K_1 \cdot \prod_{i=0}^{L-1} \exp \left\{ -\frac{a_{iL}^2}{N_0} \| \mathbf{S}_i \|^2 \right\} \cdot I_0 \left( \frac{2}{N_0} \sqrt{\langle \mathbf{r}_{iL} \mathbf{d}, \mathbf{S}_i \mathbf{a}_{iL} \rangle^2 + \langle \mathbf{r}_{iQ} \mathbf{d}, \mathbf{S}_i \mathbf{a}_{iQ} \rangle^2} \right) \cdot \prod_{i=L}^{N-1} \exp \left\{ -\frac{a_{iL}^2}{N_0} \| \mathbf{d}_i \|^2 \right\} \cdot I_0 \left( \frac{2}{N_0} \sqrt{\langle \mathbf{r}_{iL} \mathbf{d}, \mathbf{a}_{iL} \rangle^2 + \langle \mathbf{r}_{iQ} \mathbf{d}, \mathbf{a}_{iQ} \rangle^2} \right),
\]

where \( I_0(x) \) is the zero-order modified Bessel function of the first kind and \( K_1 \) is given as

\[
K_1 = \frac{1}{(\pi N_0)^N} \prod_{i=0}^{L-1} \exp \left\{ -\frac{1}{N_0} \left( \| \mathbf{r}_{iL} \mathbf{d} \|^2 + \| \mathbf{r}_{iQ} \mathbf{d} \|^2 \right) \right\}.
\]

Averaging over all the possible random data sequences \( \mathbf{d} \) yields

\[
p(\mathbf{r}|\mu) = M^{-(N-L)} K_1 \cdot \prod_{i=0}^{L-1} \exp \left\{ -\frac{a_{iL}^2}{N_0} \| \mathbf{S}_i \|^2 \right\} \cdot I_0 \left( \frac{2}{N_0} \sqrt{\langle \mathbf{r}_{iL} \mathbf{d}, \mathbf{a}_{iL} \rangle^2 + \langle \mathbf{r}_{iQ} \mathbf{d}, \mathbf{a}_{iQ} \rangle^2} \right) \cdot \prod_{i=L}^{N-1} \exp \left\{ -\frac{a_{iL}^2}{N_0} \| \mathbf{d}_i \|^2 \right\} \cdot I_0 \left( \frac{2}{N_0} \sqrt{\langle \mathbf{r}_{iL} \mathbf{d}, \mathbf{a}_{iL} \rangle^2 + \langle \mathbf{r}_{iQ} \mathbf{d}, \mathbf{a}_{iQ} \rangle^2} \right),
\]

Similarly to [4] and [5], we make use of the following equality

\[
\sum_{i=0}^{L-1} \prod_{i=L}^{N-1} \exp \left\{ -\frac{a_{iL}^2}{N_0} \| \mathbf{d}_i \|^2 \right\} = \prod_{i=0}^{L-1} \sum_{j=1}^{M} \exp \left\{ -\frac{a_{iL}^2}{N_0} \| \mathbf{W}_j \|^2 \right\} \cdot I_0 \left( \frac{2}{N_0} \sqrt{\langle \mathbf{r}_{iL} \mathbf{d}, \mathbf{a}_{iL} \rangle^2 + \langle \mathbf{r}_{iQ} \mathbf{d}, \mathbf{a}_{iQ} \rangle^2} \right),
\]

where \( \mathbf{W}_j \) is a random vector.

Defining a second constant \( K_2 \) as

\[
K_2 = \prod_{i=0}^{L-1} \sum_{j=1}^{M} \exp \left\{ -\frac{a_{iL}^2}{N_0} \| \mathbf{W}_j \|^2 \right\} \cdot I_0 \left( \frac{2}{N_0} \sqrt{\langle \mathbf{r}_{iL} \mathbf{d}, \mathbf{a}_{iL} \rangle^2 + \langle \mathbf{r}_{iQ} \mathbf{d}, \mathbf{a}_{iQ} \rangle^2} \right),
\]

we note that both \( K_1 \) and \( K_2 \) are independent of \( \mu \). Making use of this in (5) we can equivalently maximize \( \ln[p(\mathbf{r}|\mu)/(M^{-(N-L)}K_1K_2)] \) which yields
\[ \Lambda_{NC}^{\text{ML}}(\mu) \]

\[ \sum_{i=0}^{l-1} \left( - \frac{a_{i+\mu}^2}{N_0} \right) \sum_{i=0}^{l-1} \ln \left( I_0 \left[ \frac{2}{N_0} \sqrt{r_{i+\mu} S_i a_{i+\mu}^2 + r_{i+\mu} Q_i S_i a_{i+\mu}^2} \right] \right) \]

\[ - \sum_{i=0}^{l-1} \ln \left( \sum_{j=1}^{M} \exp \left\{ - \frac{a_{i+\mu}^2 E_j}{N_0} \right\} \right) \]

\[ + \frac{2}{N_0} \right) \]

which is the ML rule for determining \( \mu \) with noncoherent demodulation. In (8), \( E_j \) is defined as \( \| W_j \|^2 \).

Here observe that the second term in (8) is the correlation rule, similar to [4]. As expected, in case of no fading, i.e. \( a_i = 1 \), \( i = 0 \cdots N-1 \), (8) simplifies to the ML metric derived by Lui and Tan [4] for the case of noncoherent phase demodulation.

### III. HIGH-SNR APPROXIMATION

Using an approach similar to [4], we can obtain the high-SNR ML rule approximation:

\[ \Lambda_{h_{SNR}}^{\text{ML}}(\mu) \]

\[ \sum_{i=0}^{l-1} \left( - \frac{a_{i+\mu}^2}{2} \right) \sum_{i=0}^{l-1} \ln \left( I_0 \left[ \frac{2}{N_0} \sqrt{r_{i+\mu} S_i a_{i+\mu}^2 + r_{i+\mu} Q_i S_i a_{i+\mu}^2} \right] \right) \]

\[ - \sqrt{r_{i+\mu} S_i a_{i+\mu}^2 + r_{i+\mu} Q_i S_i a_{i+\mu}^2} \]

\[ - \sqrt{r_{i+\mu} S_i W_j a_{i+\mu}^2 + r_{i+\mu} Q_i W_j a_{i+\mu}^2} \]

\[ + \frac{a_{i+\mu}^2 E_{j(i)}}{2}, \]

where

\[ j(i) \]

\[ \max_{1 \leq j \leq M} \left[ \sqrt{r_{i+\mu} S_i W_j a_{i+\mu}^2 + r_{i+\mu} Q_i W_j a_{i+\mu}^2} \right] \]

\[ - \frac{a_{i+\mu}^2 E_j}{2}. \]

Note that the second term of (9) is the high-SNR correlation rule. The high-SNR ML rule (9) is substantially easier to implement than the exact ML rule (8) and, as our simulations indicate, provides similar performance for a wide range of SNRs. As expected, in case of no fading, (9) simplifies to the expression derived by Lui and Tan [4].

### IV. EXPERIMENTAL RESULTS

As noted in [4], analytical evaluation of the ML rules does not appear to be feasible, thus we resorted to computer simulations in order to evaluate and compare performance of the ML rule (8) with that of the high-SNR ML rule approximation (9) and the correlation rule (the second term of (9)). We should note that in case of noncoherent demodulation, the complexity of evaluating ML metric is high since it involves evaluating transcendental functions. We found, however, that when the exact ML rule (8) is used instead of the correlation rule, performance gains are substantial. We also found that by using the high-SNR rule of (9) we can achieve much better performance than by using the correlation rule but with significantly less computational effort than when using the exact ML metric. At higher SNRs, as expected, performance of the high-SNR rule approaches that of the exact ML rule.

In order to evaluate performance, we chose to simulate transmission of data using 4-ary and 8-ary FSK. To be consistent with the previous work, we considered examples from Lui and Tan [4]. For 4-ary FSK case we assume a 16-symbol-long frame with a synchronization pattern consisting of 4 distinct synchronization symbols followed by random data while for 8-ary FSK we employ an 80-symbol frame with an 8 distinct-symbol synchronization pattern followed by random data. In addition, because of our interest in transmission of compressed video over noisy channels, we chose to do simulations with the MPEG-4 start code prefix, a 24-bit pattern consisting of 23 “0”s followed by a “1” as the synchronization pattern. Thus, for 8-ary FSK the synchronization sequence was chosen to be 8 symbols. The total frame length was chosen to be 240 bits, or 30 symbols for 8-ary FSK.

The results for 4-ary FSK, noncoherent phase demodulation under fading conditions are shown in Fig. 1(a). As can be seen from the figure, the ML rule results in substantially better synchronization performance than the correlation rule, particularly at SNRs above 0 dB. As Fig. 1(a) shows, the ML rule can determine the beginning of the frame denoted by the synchronization pattern in 35% of cases at an SNR of 0 dB while the correlation rule achieves the same performance level at SNR of approximately 6 dB. Fig. 1(b) presents results of noncoherent detection of the 8-ary FSK signal in AWGN. While the
results for the ML rule and the approximate high-SNR ML rule are similar to those in [4] differing only due to the fact that we did not allow the synchronization pattern to be repeated in the data portion of the frame, we found the performance of the correlation rule to be substantially better than indicated in [4] for this case. As a result, at the SNR of 9 dB synchronization performance of 100% can be achieved.

For the case of 8-ary FSK, with noncoherent phase demodulation under fading conditions, results are shown in Fig. 2(a) (with synchronization pattern consisting of 8 distinct symbols) and Fig. 2(b) (with an 8-symbol synchronization pattern corresponding to the MPEG-4 start code prefix). Again, both figures show that by using ML rule we can achieve substantially better synchronization performance than when using the correlation rule. ML rule is capable of synchronizing in 40% of the cases when distinct synchronization pattern is used, as in Fig. 2(a), at signal-to-noise ratio of 0 dB while the correlation rule achieves the same performance level at SNR of about 6 dB.

Finally, it is of interest to compare performance of different synchronization patterns under the same conditions, as in 8-ary FSK when distinct-symbol and MPEG-4 synchronization patterns are used. As we found, the choice of synchronization pattern affects performance significantly. Comparing the results in Fig. 2(a) and Fig. 2(b) it can be observed that when 8 distinct symbols are used as a synchronization pattern, the ML rule is able to determine the start of the frame in 75% of cases at SNR of 3 dB. However, when the MPEG-4 synchronization pattern is used, in order to achieve 75% synchronization performance, an SNR of 6 dB is required. This result, while not intuitive, is not unexpected, as certain desirable correlation properties are required of the selected synchronization patterns [1, 3] which are not present in the MPEG-4 pattern.

We should also note that while for the AWGN channel the high-SNR rule performs nearly as well as the exact ML rule, the difference in performance is more substantial in case of fading. Therefore, there is an incentive to use the exact ML rule, if possible, if the signal is affected by fading.

V. CONCLUSIONS

In this paper we derived the ML rule and approximate high-SNR ML rule for detecting frame synchronization patterns for the flat Rayleigh fading channel with noncoherent phase M-ary signaling. We found that if channel state information is known, detection of the frame synchronization patterns can be substantially improved compared to the correlation rule. We also found that the choice of the synchronization pattern greatly affects performance of the synchronizer.

References


Fig. 1: Performance of the ML, high-SNR ML and correlation rules for distinct-symbol synchronization pattern detection.

(a) 4-FSK, Rayleigh fading channel.  
(b) 8-FSK, AWGN channel.

Fig. 2: Performance of the ML, high-SNR ML and correlation rules for synchronization pattern detection. Results are shown for 8-FSK on the Rayleigh fading channel.

(a) Distinct-symbol synchronization pattern.  
(b) MPEG-4 synchronization pattern.