Detection of a Known Number of Aperiodically Embedded Synchronization Patterns

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Abstract — In this work we consider the problem of locating aperiodically embedded synchronization patterns in random data which is subject to additive white Gaussian noise. We derive an optimum detection rule and show experimental results comparing the performance of the derived maximum-likelihood rule to a correlation rule. It is shown that the well-known correlation rule performs substantially worse than the maximum-likelihood rule.

I. INTRODUCTION

A substantial amount of work has been performed in the area of locating periodically embedded known synchronization patterns. In [1] Barker considered the problem of locating a known synchronization pattern in random data and proposed the correlation rule. Later, Massey [2] derived the maximum likelihood (ML) rule for detecting periodically embedded synchronization patterns for the AWGN channel with coherent BPSK modulation. He has shown that performance of the ML rule is substantially better than that of the correlation rule, earlier believed to be optimal. He also derived low-SNR and high-SNR approximations to the ML rule. Nielsen [3] analyzed Massey’s results to obtain performance limits for both the ML and the correlation rules. Lui and Tan [4] extended Massey’s derivation to include M-ary communication systems. Robertson [5] further extended Lui and Tan’s derivation to include the Rayleigh fading channel when M-ary coherent phase signaling is used.

The problem of locating aperiodically embedded synchronization patterns has been studied in substantially less detail. Massey [2] noted that deriving the ML rule for locating aperiodically embedded synchronization patterns would be interesting, but did not attack this problem. Recently, Eck, et al. [6] approached this problem from a detection theory standpoint. They derived a metric consisting of Massey’s ML metric and an additional correction term, and then determined experimentally that Massey’s metric is sufficient. In this paper we extend Massey’s ML rule derivation to the case of aperiodically embedded synchronization pattern and show that the ML rule for this case is actually the same as that derived by Massey in [2]. Since exact analysis of the performance of the synchronization rule does not appear to be feasible [4] we resort to computer simulations for performance evaluation. The motivation behind this work is error resilience and recovery in applications such as MPEG-4 [7] video transmission which can benefit from improved detection of start codes.

II. DERIVATION OF THE MAXIMUM LIKELIHOOD RULE

We consider an AWGN channel with data transmitted using BPSK modulation. A frame of varying length, N, consists of an a priori known synchronization pattern of length L bits \((s_0, s_1, \ldots, s_{L-1})\), represented by \(\mathbf{S}\), followed by random data bits \((d_L, d_{L+1}, \ldots, d_{N-1})\) represented by \(\mathbf{D}\). A single frame is represented by concatenating \(\mathbf{SD} = (s_0, s_1, \ldots, s_{L-1}, d_L, d_{L+1}, \ldots, d_{N-1})\). In video coding applications \(\mathbf{S}\) would represent a synchronization sequence such as a start code, while \(\mathbf{D}\) would represent overhead information and coded data. In order to simplify analysis, we assume that all random data bits within a frame are equally likely. Typically, no restriction is made on the possibility of the synchronization pattern repeated in data. However, it is well-known that for practical coder implementations such as MPEG-4, a synchronization pattern is not allowed to reappear within
a frame. We accounted for this fact in simulations by ensuring that no synchronization pattern appears in the random data portion of the frame.

A transmitted sequence of bits is received as \( \mathbf{r} \) which represents the bits corrupted by additive white Gaussian noise. The contribution of AWGN to each transmitted frame is \( \pi = (n_0, n_1, \cdots, n_{N-1}) \). The components \( n_i \) of \( \pi \) are assumed to be statistically independent Gaussian random variables with zero mean and variance \( \frac{\sigma^2}{N_0} \), where \( N_0 \) is the one-sided power spectral density. In the absence of noise each received digit would have values of \( \pm \sqrt{E} \) where \( E \) is the signal energy. All subscripts on received digits will be taken modulo \( N \) for a given frame of length \( N \), i.e. bit \( r_0 \) will follow digit \( r_{N-1} \).

Following Massey’s derivation for periodically embedded synchronization patterns [2] if \( \mathbf{r} = (\rho_0, \rho_1, \cdots, \rho_{N-1}) \) denotes the actual value of the random vector \( \mathbf{r} \), the optimum decision rule in the sense of maximizing the probability of correctly locating the synchronization pattern within a frame is to choose as the estimate of \( m \) the value of \( \mu \), \( 0 \leq \mu < N \), for which the metric \( P(m = \mu \mid \mathbf{r} = \mathbf{p}, n = N) \) is maximized. The synchronization pattern is a priori equally likely to begin in any of the \( N \) positions of \( \mathbf{r} \) within a frame and therefore \( P(m = \mu \mid n = N) = \frac{1}{N} \) for all \( \mu \). Additionally, \( p(\mathbf{r} \mid n) \) does not depend on \( \mu \). Thus, by the mixed Bayes’ rule, we can equivalently use \( p(\mathbf{r} \mid m = \mu, n = N) \) as the metric to be maximized. Letting \( \delta = (\delta_L, \delta_{L+1}, \cdots, \delta_{N-1}) \), where \( \delta_i \in \{+1, -1\} \), denote a possible value of the random data vector \( \mathbf{d} \) and taking into account the fact that noise \( \pi \) is assumed to be Gaussian, we have

\[
p(\mathbf{r} \mid \mathbf{d} = \delta, m = \mu, n = N) = \left( \frac{1}{\sqrt{2\pi N_0}} \right)^N \cdot \prod_{i=0}^{L-1} e^{-\frac{(\sqrt{x_i + \bar{x}_i})^2}{N_0}} \prod_{j=L}^{I} e^{-\frac{\sqrt{x_j + \bar{x}_j}}{N_0}}.
\]

This expression may be rewritten as

\[
- \left( \frac{1}{\sqrt{\pi N_0}} \right)^N \prod_{i=0}^{L-1} e^{-\frac{2\bar{x}_i}{N_0} + \frac{2x_i + \bar{x}_i}{N_0}} \prod_{i=0}^{I} e^{-\frac{2\bar{x}_j}{N_0} + \frac{2x_j + \bar{x}_j}{N_0}}
\]

\[
= \left( \frac{1}{\sqrt{\pi N_0}} \right)^N \cdot \prod_{i=0}^{L-1} e^{-\frac{\sqrt{2x_i + \bar{x}_i}}{N_0}} \prod_{j=L}^{I} e^{-\frac{\sqrt{2x_j + \bar{x}_j}}{N_0}}
\]

\[
= \left( \frac{1}{\sqrt{\pi N_0}} \right)^N \cdot \prod_{i=0}^{L-1} e^{-\frac{\sqrt{2x_i + \bar{x}_i}}{N_0}} \prod_{j=L}^{I} e^{-\frac{\sqrt{2x_j + \bar{x}_j}}{N_0}}.
\]

The probability

\[
P(\mathbf{d} = \delta) = \sum_{m=0}^{N} P(\mathbf{d} = \delta \mid n = N) P(n = N)
\]

\[
= \sum_{m=0}^{N} 2^{-(N-L)} P(N)
\]

\[
= K,
\]

where \( K \) is a constant. Thus, the metric to be maximized is written as

\[
p(\mathbf{r} \mid m = \mu, n = N)
\]

\[
= \sum_{\mathbf{d}} p(\mathbf{r} \mid \mathbf{d} = \delta, m = \mu, n = N) P(\mathbf{d} = \delta)
\]

\[
= K \sum_{\mathbf{d}} p(\mathbf{r} \mid \mathbf{d} = \delta, m = \mu, n = N)
\]

\[
= K \cdot \left( \frac{1}{\sqrt{\pi N_0}} \right)^N \cdot \prod_{i=0}^{L-1} e^{-\frac{\sqrt{2x_i + \bar{x}_i}}{N_0}} \prod_{j=L}^{I} e^{-\frac{\sqrt{2x_j + \bar{x}_j}}{N_0}}.
\]

Rather than employing the full metric, (4) can be simplified with no degradation in resulting performance. If we define

\[
C_N \triangleq K \cdot \left( \frac{1}{\sqrt{\pi N_0}} \right)^N \cdot \prod_{i=0}^{L-1} e^{-\frac{\sqrt{2x_i + \bar{x}_i}}{N_0}} \prod_{j=L}^{I} e^{-\frac{\sqrt{2x_j + \bar{x}_j}}{N_0}}.
\]

we realize that (5) is a constant due to the modulo \( N \) addition of indices. Now we can define a metric

\[
M_1(\mu) = p(\mathbf{r} \mid m = \mu, n = N) / C_N
\]

\[
= \left( \frac{1}{\sqrt{\pi N_0}} \right)^N \cdot \prod_{i=0}^{L-1} e^{-\frac{\sqrt{2x_i + \bar{x}_i}}{N_0}} \prod_{j=L}^{I} e^{-\frac{\sqrt{2x_j + \bar{x}_j}}{N_0}}.
\]

At this point metric \( M_1(\mu) \) no longer depends on the frame length, \( N \). Equivalently, we can consider \( \ln(M_1) \)
as our metric. Dropping factors independent of $\mu$ and rearranging the resulting expression, the simplified metric $\Lambda(\mu)$ becomes

$$\Lambda(\mu) = \sum_{j=0}^{L-1} \rho_j \mu^{a_j} - \frac{N_0}{2\sqrt{E}} \sum_{k=0}^{L-1} \ln \cosh \frac{2\sqrt{E}\rho_k + \mu}{N_0}. (7)$$

We should note that metric (7) is identical to the one derived by Massey for detecting a periodically embedded synchronization pattern (and consists of a correlation term and a correction term). However, in order to detect where the synchronization patterns were embedded, knowledge of the number of synchronization patterns transmitted is needed since the metric no longer has a single maximum but rather has a number of maxima equal to the number of transmitted synchronization patterns in the bitstream. We should note that Eck, et al. [6] through a different approach found a metric equal to the Massey’s metric plus additional correction term that depends on the distribution of the possible frame lengths. However, their simulations indicated that this additional correction does not affect detection performance.

III. EXPERIMENTAL RESULTS

Both Massey [2] and Lui and Tan [4] noted that analytical evaluation of performance of the ML rule does not appear to be feasible, thus we resorted to computer simulations in order to evaluate and compare performance of the ML rule of (7) with that of the well-known correlation rule, represented by the first term of (7).

In order to evaluate performance, we simulated transmission of data in equiprobable frames of length 20 or 30 bits. To be consistent with the previous work, we considered the 7-bit Barker sequence $\{1, -1, 1, 1, -1, -1, -1\}$ as our synchronization pattern. Thus, for a 20-bit frame, the 7-bit Barker sequence is followed by 13 bits of random data. We also assume knowledge of the number of synchronization patterns present in the transmitted bitstream, as noted earlier. The derived ML metric (7) is computed for all possible $\mu$; and the synchronization patterns are assumed to be present at positions indicated by the maxima of the metric. The number of maxima considered is equal to the number of synchronization patterns present in the bitstream.

The results for this case are shown in Figure 1. As can be seen from the figure, at all SNRs of interest using ML rule (7) results in substantially better synchronization performance over using the correlation rule. The ML rule can determine synchronization patterns in 100% of the cases at an SNR of 4.5 dB, while the correlation rule achieves similar performance, of 99.3% correctly detected synchronization patterns, at an SNR of 11 dB. At an SNR of 1.5 dB the correlation rule can detect only 75.9% of the synchronization patterns while the ML rule can detect 93.1%. This indicates that detection performance gains over the correlation rule which are similar to or even better than the case of the periodically embedded synchronization patterns, may also be achieved by using the ML metric when the synchronization patterns are aperiodically embedded.

Also of practical interest is the case when non-equiprobable frames are transmitted. As before, we used the Barker sequence as our synchronization pattern and assumed knowledge of the number of synchronization patterns present in the transmitted bitstream. For simulations we again used 20-bit and 30-bit frames, occurring with probabilities 0.2 and 0.8, respectively.

The results for this case are shown in Figure 2. It should be noted that we expected slightly degraded performance compared to detecting synchronization patterns for equiprobable frames, since our metric (7) does not take into account the probability of frame occurrences. As the figure demonstrates, synchronization performance is slightly below that of the equiprobable frames. We can detect 99% of synchronization patterns at SNR of 4.5 dB using the ML rule, while the correlation rule can only detect 97% of synchronization patterns at SNR of 11 dB. Performance loss for the ML rule (7) compared to detection of synchronization patterns for equiprobable frames varies for different SNRs, reaching a maximum of approximately 5% for SNRs of 1.5 dB and -1 dB. Interestingly, performance degradation for the correlation rule, known to be non-optimal, is more significant, reaching a maximum 8% at SNR of 1.5 dB. These results point out possible performance gains may be achieved with a metric that takes into account probabilities of frame occurrences.

IV. CONCLUSIONS

In this paper we derived the ML rule for detecting aperiodically embedded frame synchronization patterns when BPSK signaling is used. We showed that it is identical to the ML rule for periodically embedded synchronization patterns, under the assumption that the number of synchronization patterns in the data is known. We found that substantial performance gains in synchronization pattern detection may be achieved by using the ML rule as compared to the well-known correlation rule. Future work will be directed toward deriving metrics for
Figure 1: Performance of the ML and correlation rules for 7-bit Barker sequence synchronization pattern detection.

detection of an unknown number of embedded synchronization patterns by taking into account probabilities of frame occurrences.

References


Figure 2: Performance of the ML and correlation rules for 7-bit Barker sequence synchronization pattern detection (non-equiprobable frames, \( P(20 \text{ bit frame}) = 0.2, P(30 \text{ bit frame}) = 0.8 \)).