THREE MANAGEMENT POLICIES FOR A RESOURCE WITH PARTITION CONSTRAINTS

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Abstract
Management of a bufferless resource is considered under non-homogeneous demand consisting of one-unit and two-unit requests. Two-unit requests can be served only by a given partition of the resource. Three simple admission policies are evaluated with regard to revenue generation. One policy involves no admission control and two policies involve trunk reservation. A limiting regime in which demand and capacity increase in proportion is considered. It is shown that each policy is asymptotically optimal for a certain range of parameters. Limiting dynamical behavior is obtained via a theory developed by Hunt and Kurtz. The results also point out the remarkable effect of partition constraints.

Keywords: Resource management; partition constraints; loss networks; multirate networks; admission control; trunk reservation; heavy traffic; time-scale separation

AMS 1991 Subject Classification: Primary 60K30; Secondary 90B22; 68M20; 93E20

1. Introduction
This paper investigates effective control policies for a bufferless resource that operates under non-homogeneous dynamic demand. The demand consists of requests of two different types, categorized by the number of resource units required for service. Management of the resource is subject to partition constraints: requests of each type can be serviced only by a block from an associated partition of the resource. We consider in detail the case when one type requires twice as many resource units as the other.

Partition constraints typically arise in time-division-multiplexed multirate communication systems, owing to certain operational limitations. An instance of the problem addressed in this paper arose in the global system for mobile communication (GSM). The system accommodates full-rate users, each of which requires a full-time-slot, as well as half-rate users, each of which requires a half-time-slot. A pair of half-time-slots can accommodate a full-rate user only if they form a full-time-slot, so the collection of all such pairs constitutes a resource partition for full-rate users.

We consider the following stochastic setting. Let \( \lambda_f \), \( \lambda_h \), and \( C \) be fixed positive numbers, and let \( \gamma \) be a positive scaling factor. There are two types of calls denoted by full-rate calls and half-rate calls. Full-rate calls arrive according to a Poisson process of rate \( \gamma \lambda_f \) and half-rate calls arrive according to a Poisson process of rate \( \gamma \lambda_h \). The two arrival processes are mutually independent.

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independent. The total number of available slots is \( \lfloor \gamma C \rfloor \). A slot can be assigned either one full-rate call or at most two half-rate calls. There is no buffering, thus a call is blocked if it is not assigned a slot immediately upon its arrival. Blocked calls cannot be assigned later, and have no effect on the future evolution of the system. A slot is said to be occupied if it is assigned one full-rate call or two half-rate calls, partially occupied if it is assigned one half-rate call, or idle otherwise. A full-rate call is blocked if upon its arrival there are no idle slots, and a half-rate call is blocked if upon its arrival all slots are occupied. Calls can also be blocked in other circumstances depending on the admission policy, which is a decision mechanism to accept or reject an arriving call. For efficient use of capacity, an accepted half-rate call is assigned an idle slot only if there are no partially occupied slots at the time of its arrival.

Each accepted call remains in the system for the duration of its holding time, during which it maintains the same slot assignment. The holding time of a call is an exponentially distributed random variable with unit mean, independent of its type and the history of the system prior to its arrival. If accepted, each full-rate call generates revenue at rate \( r_f \) and each half-rate call generates revenue at rate \( r_h \) throughout the holding time.

A similar stochastic setting in which calls require either one or six resource units has been a subject of considerable interest in the context of ISDN communication systems. In that setting Ramaswami and Rao (1985) studied approximate call blocking probabilities in the absence of admission control. Reiman and Schmitt (1994) considered trunk reservation type admission policies as well, and studied effective methods to determine the blocking probabilities in the case when call types have vastly different time scales. Ross and Tsang (1989) focused on efficient methods to determine admission policies that maximize resource utilization.

In this paper effectiveness of an admission policy is measured with the revenue generated in the long term. We examine three policies which have desirable features such as simplicity and robustness to traffic parameters. These policies are evaluated in a limiting regime that corresponds to arbitrarily large values of the scaling factor \( \gamma \), and it is shown that asymptotically each policy generates revenue at maximum rate for certain values of the parameters \((r_f, r_h)\). In addition to equilibrium properties, explicit descriptions of the transient system behavior are also obtained.

The first policy considered in the paper is trunk reservation for full-rate calls, under which a full-rate call is accepted whenever there is an idle slot, whereas a half-rate call is accepted only if the number of idle slots is larger than a reservation threshold \( T(\gamma) \). Note that acceptance of a half-rate call does not depend on the availability of partially occupied slots. The reservation threshold grows unboundedly with \( \gamma \) (i.e. \( \lim_{\gamma \to \infty} T(\gamma) = \infty \)), however slower than \( \gamma \) itself (i.e. \( \lim_{\gamma \to \infty} T(\gamma)/\gamma = 0 \)). The second policy, trunk reservation for half-rate calls, prescribes accepting a half-rate call unless all slots are occupied, and accepting a full-rate call only if the number of idle slots is larger than \( T(\gamma) \). Finally we consider complete sharing under which no admission control is exercised, so that a call is accepted if there is enough capacity to accommodate it.

Trunk reservation has been studied extensively in stochastic settings that do not involve partition constraints. Miller (1969) showed that under homogeneous traffic a trunk reservation policy maximizes the rate of revenue generation among non-anticipative admission policies. If either the request size or the mean holding time varies with call type, such a conclusion holds in a limiting regime similar to the one considered here, as established by Hunt and Laws (1997). The work of Hunt and Laws (1997) is closely related to the work of Bean et al. (1995, 1997) which studies the limiting behavior of trunk reservation. All three papers are based on the theory developed in Hunt and Kurtz (1994) which provides a detailed description of
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the limiting system dynamics, particularly on boundaries along which the system statistics are discontinuous. In the context of the present paper such boundaries arise as a result of depletion of idle or partially occupied slots, and our analysis also is based on Hunt and Kurtz (1994).

In the remainder of this section we state the main results of the paper, starting with some essential definitions. For each \( t \geq 0 \) let the random vector \( X_t = (X_t(1), X_t(2), X_t(3)) \) be defined as

\[
X_t(1) = \text{number of slots occupied by full-rate calls at time } t, \\
X_t(2) = \text{number of slots occupied by (two) half-rate calls at time } t, \\
X_t(3) = \text{number of partially occupied slots at time } t.
\]

and set \( X_\gamma t = (X_t(1)/\gamma, X_t(2)/\gamma, X_t(3)/\gamma) \). An initial condition and an admission policy determine the random processes \( X = (X_t: t \geq 0) \) and \( X_\gamma = (X_\gamma t: t \geq 0) \). The long-term average rate of revenue generated by an admission policy \( \Pi_1 \), \( \Pi_2 \), is expressed as

\[
\frac{J}{\Pi_1} = \limsup_{T \to \infty} \mathbb{E}\left[ \frac{1}{T} \int_0^T \left( r_f X_t(1) + r_h (2 X_t(2) + X_t(3)) \right) dt \right].
\]

Under each of the three admission policies of interest, the process \( X_\gamma \) is ergodic and \( X_\gamma \infty \) denotes the equilibrium random vector. Given real numbers \( a, b \) let \( a \wedge b \) denote the smaller of \( a \) and \( b \), and define \( x^* = (C \wedge \lambda f, (C - x^* (2)) \wedge \lambda h / 2, 0) \).

The main contribution of the paper has two aspects. First, asymptotic optimality of the admission policies considered is established by the following three theorems. Here it is remarkable that complete sharing asymptotically achieves full priority for half-rate calls without the need for trunk reservation. Second, a methodical approach is shown to identify the limiting dynamical behavior via the theory of Hunt and Kurtz (1994).

Theorem 1.1. Under trunk reservation for full-rate calls (TRF)

\[
\lim_{\gamma \to \infty} X_\gamma \infty = x^* \text{ in probability. In particular if } r_f \geq 2 r_h \text{ then for any admission policy } \Pi_1 \text{ we have } \\
\sup_{\gamma} \mathbb{E}\left[ \frac{J}{\Pi_1} / \gamma \right] \leq \lim_{\gamma \to \infty} \frac{J}{\Pi_1} \infty = r_f x^* (1) + r_h (2 x^* (2) + x^* (3)).
\]

Theorem 1.2. Under trunk reservation for half-rate calls (TRH)

\[
\lim_{\gamma \to \infty} X_\gamma \infty = x^* \text{ in probability. In particular if } r_f \leq 2 r_h \text{ then for any admission policy } \Pi_1 \text{ we have } \\
\sup_{\gamma} \mathbb{E}\left[ \frac{J}{\Pi_1} / \gamma \right] \leq \lim_{\gamma \to \infty} \frac{J}{\Pi_1} \infty = r_f x^* (1) + r_h (2 x^* (2) + x^* (3)).
\]

Theorem 1.3. Under complete sharing (CS)

\[
\lim_{\gamma \to \infty} X_\gamma \infty = x^* \text{ in probability. In particular if } r_f \leq 2 r_h \text{ then for any admission policy } \Pi_1 \text{ we have } \\
\sup_{\gamma} \mathbb{E}\left[ \frac{J}{\Pi_1} / \gamma \right] \leq \lim_{\gamma \to \infty} \frac{J}{\Pi_1} \infty = r_f x^* (1) + r_h (2 x^* (2) + x^* (3)).
\]
FIGURE 1: Typical trajectories that approximate the transient behavior of the system under (a) trunk reservation for full-rate calls, (b) trunk reservation for half-rate calls, and (c) complete sharing, in the case \( C = \lambda_f = \lambda_h / 2 = 3. 

We now briefly comment on the theorems. The vector \( x^\ast \) (respectively the vector \( x^\ast \)) characterizes an operating point at which the available capacity is used primarily to accommodate full-rate (half-rate) calls, leaving only the excess capacity for half-rate (full-rate) calls. Moreover half-rate calls are almost perfectly packed so that there is only a marginal number of partially occupied slots. If \( r_f \geq 2r_h \) (\( r_f \leq 2r_h \)) then such an operating point is almost optimal, and by Theorem 1.1 (Theorem 1.2) trunk reservation achieves asymptotic optimality by maintaining the system sufficiently close to it. By Theorem 1.3 the uncontrolled system tends to evolve around the same operating point as the system under the TRH policy, so that complete sharing is also asymptotically optimal if \( r_f \leq 2r_h \).

The partition constraint has a remarkable effect on the natural evolution of the system, as pointed out by Theorem 1.3: in the absence of partition constraints, it follows from Kelly (1986) that complete sharing results in limiting blocking probabilities of \( (1 - q^2) \) and \( (1 - q) \) for full-rate and half-rate calls respectively, where \( q \) denotes the positive root of \( \lambda_f q^2 + (\lambda_h / 2) q - C = 0 \) and \( (\cdot) \) denotes max \( (\cdot), 0 \). When the partition constraint is imposed, however, full-rate calls may experience a disproportionately large blocking probability, to the extent that they may be totally locked out of the system in the large \( \gamma \) limit.

We finally comment on the transient behavior of the system under the three admission policies. Figure 1 illustrates trajectories that well-approximate the process \( X^\gamma \) for large values of \( \gamma \), in the case \( C = \lambda_f = \lambda_h / 2 = 3 \) and \( X^\gamma_0 = 0 \). An intuitive interpretation of these
Appendix.

Theorems 1.1, 1.2, and 1.3 respectively. Proofs of some auxiliary results are collected in the

advantage is significant enough so that eventually half-rate calls monopolize the entire system.

smaller rate than full-rate calls do; in turn half-rate calls have an inherent advantage. This

the number of partially occupied slots is marginal, half-rate calls release idle slots at a much

arrivals. Under complete sharing, full-rate and half-rate arrivals compete for idle slots. Since

increasing the number of such slots to $O(\gamma)$ of half-rate departures contribute to the number of partially occupied slots, thereby

calls experience virtually no blocking. In contrast, under trunk reservation for full-rate calls, a

full-rate departure immediately enables admission of two half-rate arrivals, in turn half-rate

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so that for $t \in \mu(X_0, \gamma)$, the mapping $\nu_\gamma \circ V_t$ establishes that the limit trajectories conform to certain ordinary differential equations.

Let the set of integers and the set of non-negative integers respectively. Since $\nu_\gamma$ is compact by Proposition 2.3, in which case it is assigned an idle slot if and only if

$$\lambda \nu_\gamma(x) > \infty,$$

for all $x$. Since $\nu_\gamma$ is compact, the proof follows.
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\[ \lambda_f \]

\[ \lambda_h \]

\[ \pi \]

\[ \nu \]

\[ \nu(\cdot) = \int_0^t \pi(\cdot) \, ds, \quad t \geq 0, \quad B \in \mathcal{B}(E), \]

where \( \pi \) is an equilibrium distribution for a Markov process \( Y_{x\nu} = (Y_{x\nu}(t): t \geq 0) \) that takes values in \( E \) and has transition rates given by

- \( Y_{x\nu} \leftarrow \begin{array}{ll}
    Y_{x\nu} + (0, -1, -1) & \text{at rate } \lambda_f \{ Y_{x\nu} \in A_1 \}
    \\
    Y_{x\nu} + (0, +1, +1) & \text{at rate } x_1(1)
    \\
    Y_{x\nu} + (-1, 0, 0) & \text{at rate } \lambda_h \{ Y_{x\nu} \in A_2 \}
    \\
    Y_{x\nu} + (+1, 0, 0) & \text{at rate } 2x_2(1)
    \\
    Y_{x\nu} + (+1, -1, -1) & \text{at rate } \lambda_h \{ Y_{x\nu}(1) > 0 \}
    \\
    Y_{x\nu} + (-1, +1, +1) & \text{at rate } x_3(1)
\end{array} \]

Here and in the rest of the paper it is understood that \( \pm \infty + k = \pm \infty \) for all \( k \in \mathbb{Z} \). In particular, \((Y_{x\nu}(1), Y_{x\nu}(2), -\infty)\) and \((Y_{x\nu}(1), +\infty, Y_{x\nu}(3))\) are effectively two-dimensional Markov processes whose transition diagrams are illustrated by Figures 2(a) and 2(b) respectively.

The process \( Y_{x\nu} \) is reducible due to the isolated states at infinity; in turn it admits multiple equilibrium distributions. The distribution \( \pi \) is therefore some convex combination of the...
function $y$ at time $s$ changes its value. In particular the instantaneous rates of various admission and allocation of the same lemma follow by straightforward interpretation of Hunt and Kurtz (1994).

Theorem $\nu \gamma$ holds for almost all $s$ of the same lemma. The lemma is proved in the Appendix. An intuitive interpretation of the above description is as follows. For large values of $X$ the normalized system process $Y_{xs}$ approximates the localized feedback process $L_{ys}$. For small values of $X$ the feedback process takes on many different values due to an analogue of part of Ethier and Kurtz (1986) employed a construction similar to the feedback process to analyse a trunk reservation policy.

We now provide a characterization of the limit trajectory $y$. The following conditions hold for almost all $s$.

1. The condition $1$ holds for all $s$.
2. If $y_{xs}$ then $\pi_{ys}$. The collection $\pi_{ys}$, restricted to ergodic closed subsets of the state space. More formally, we adopt the following convention to represent $\pi_{ys}$.
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58. If $y_{xs}$ then $\pi_{xs}$.
implies that all regular points \( t \) of \( x \), point \( x \) for the solutions of (2.4)–(2.7). Let \( P \) Policies for a resource with partition constraints

\[ \pi \]

Proof. This section establishes an explicit representation in terms of ordinary differential equations

\[ \dot{\lambda} \]

\[ \dot{\lambda} \]

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Remark 2.1. If \( p \in \pi \) then \( \pi_p \) is ergodic. By the previous results \( \cup \pi_p \) is also ergodic. It is then shown that these conditions determine essential

\[
\lambda \circ \left( \pi (h) \circ \lambda \right) = \lambda \circ \left( \lambda \circ \pi (h) \circ \lambda \right)
\]

for any \( h \in Z \). The general argument of the following remark was used in a somewhat similar setting by M. ALANYALI.

Finally, if \( \pi \) is ergodic then the measure \( \lambda \circ \pi (h) \circ \lambda \) is ergodic.
Consider the following two cases.

i.

\[ \dot{x}_t \]

For almost all \( t \)

\[ \text{Lemma 2.5.} \]

\[ \text{lemma.} \]

ii.

\[ \dot{x}_t \]

and

\[ \text{Lemma 2.4.} \]

\[ \text{Lemma 2.7.} \]

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\[ \dot{x}_t \]

\[ \text{Proof.} \]

\[ q \]

so that

\[ \dot{x}_t \]

we may concentrate on the case when

\[ \text{implies that} \]

\[ \dot{x}_t \]

holds:

\[ \text{The following lemma, which is instrumental for the proof of Lemma 2.7, is proved in the} \]

\[ \text{proof.} \]

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Almost all $\gamma$ tend to remain in the vicinity of $S$ and for large $\lambda$, as a unique combination of distinct probability distributions. Such non-uniqueness arises from the fact that the method employed here may not identify all trajectories uniquely.

Remark 2.2. Lemmas 2.3, 2.4, 2.5, and 2.7 imply that $(\lambda, z)$, $\lambda > 0$, is ergodic. If $\lambda = 0$, then $\lambda$ is not ergodic, and no such combination exists. Otherwise, one can take $\lambda = 0$ and $z$ tends to be uniquely identified in all cases.

Theorem 1.1. Here we establish only the weaker claim that each limit trajectory $x_t$ is not ergodic, and no such limit trajectory exists. Otherwise, one can take $\lambda = 0$ and $z$ tends to be uniquely identified in all cases.

Proof. Fix $\lambda > 0$ and let $(\lambda, z)$ be a given initial condition. Then $x_t$ converges to the point $x_t \equiv \gamma$, $\lambda > 0$, and by Lemma 2.2 it is necessary that $x_t \equiv \gamma$, $\lambda > 0$, is ergodic.

The process $\lim_{t \to \infty} x_t$, $x_t$ is ergodic. If $\lambda = 0$, then $\lambda$ is not ergodic, and no such limit trajectory exists.
Proof of Theorem 1.1. Let 
\[ t^\ast = \min \{ t \in \mathbb{R}_+ : \gamma(t) > \lambda \} \]
such that 
\[ E_{\mathbb{F}}(\gamma(t)^2) < \lambda \]
and 
\[ \gamma(t)^2 \leq f(\lambda) \]
for almost all 
\[ t \in \mathbb{R}_+ \]. This completes the proof of the theorem.
applies verbatim and establishes that the sequence as in Section 2. By redefining the sets $A$ and $\pi$, transition rates given by (2.2). In effect for $x \in \mathcal{A}$, the discussion of trunk reservation for half-rate calls.

Lemma 3.1. If $x$ satisfies (2.5)–(2.7) then it is differentiable at almost all $t$ for all regular points $t$ of $x$, and (b) the process $X_t$ is an equilibrium distribution of a Markov process $\mathcal{F}t, \mathcal{S}t)$ is defined. Under trunk reservation for half-rate calls, $X_t$ is tight. The limit $X_t$ is differentiable at almost all $t$ for all regular points $t$ of $x$, and $x \in \mathcal{A}$.

FIGURE 3: Transition diagrams of (a) the process $X_t$, and (b) the process $X_t$ for $x \in \mathcal{A}$.
If \(i\) satisfies (2.8)–(2.10), the proof is completed by obtaining the probabilities for policies for a resource with partition constraints separately: \(t\) and \(\pi\).

<table>
<thead>
<tr>
<th>TABLE 2: Valid expressions for (A)</th>
<th>(Z)</th>
<th>(A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x\times{+\infty}\times{-\infty})</td>
<td>(+\times{+\infty}\times{-\infty})</td>
<td>(+\times{+\infty}\times{-\infty})</td>
</tr>
</tbody>
</table>

The representation (2.3) implies that if \(i\) is ergodic and \(\pi\) is ergodic then without loss of generality we consider the case \(xt\): if \(\lambda(\pi)\) is ergodic then

\[
\sum_{\pi} \lambda(\pi) = \lambda(\pi_0)
\]

where \(\lambda(\pi_0)\) is the initial distribution.

If \(\lambda(\pi_0)\) is ergodic and \(\pi\) is ergodic then

\[
\pi = \frac{\sum_{\pi} \lambda(\pi)}{\lambda(\pi_0)}
\]

and

\[
\lambda(\pi) = \pi_0
\]

where \(\lambda(\pi_0)\) is the initial distribution.
Lemma 3.2. Lemmas 2.3 and 2.4 respectively, except that Lemma 2.2 is replaced by Lemma 3.1.

Lemma 3.4. For almost all $t$

Lemma 3.6. If $x_t$

Lemma 3.7. A trajectory $x_t$ satisfying (2.4)–(2.7) conforms to explicit differential equations as identified by Remark 2.1 it suffices to establish the lemma for $x_t(1)$, therefore it is necessary that $x_t(2) \in \{+\infty\} \times \{+\infty\} \times A$; therefore $x_t(1) + x_t(2) = x_t(3)$.

By Remark 2.1 we may concentrate on the case $x_t(1) = x_t(2) = x_t(3)$, and the condition $x_t(3) \leq x_t(1)$ requires that $x_t(1) = x_t(2) = x_t(3)$, which, via Lemma 3.1, requires that $x_t(3) = x_t(1) = x_t(2)$.
then \( x_t(\epsilon) \) exists. Otherwise one can choose \( \gamma > 0 \) such that \( \limsup_{t \rightarrow \tau(\epsilon)} x_t \) exists. Thus, by Lemma 3.1, the condition \( h_x = \limsup_{t \rightarrow \tau} x_t \) is not ergodic for any \( \lambda_1, \lambda_2 > 0 \) such that \( h_x < \lambda_1, \lambda_2 > 0 \) only if \( \liminf_{t \rightarrow \tau} x_t = \infty \).

2: By Lemma 3.5, any \( x_t \) such that \( x_t(\epsilon) \) exists is almost all such \( x_t \) for almost all such \( x_t \) of \( x_t(\epsilon) \).
This section proves Theorem 1.3 on the asymptotic optimality of complete sharing in the representation (2.3) implies that $\pi \in \{ + \infty \} \times \{ - \infty \}$ for all $\gamma, \nu > 0$. The limit of a weakly convergent subsequence of $\nu, \gamma > 0$ can be taken as in Table 3.

The proof of Theorem 1.1 is given in the Appendix. It follows from the representation (2.3) that $\pi$ satisfies (2.4)–(2.7). Here $X_1 \geq 0$, $X_3 = 3$, and $X_2 = 2$. The proof is completed in the Appendix.
Lemma 4.2. For almost all $t$

The lemma now follows by substituting the expressions for probabilities in (2.8)–(2.10).

Lemma 4.3. For almost all $t$
If \( C_0 \) is not ergodic for any \( \lambda \) such that \( \lambda \geq 0 \) only if \( \lambda = \infty \) and \( \lambda = \infty \).

Lemma 4.4.

The following lemma is proved in the Appendix.

If \( C_0 \) is not ergodic for any \( \lambda \) such that \( \lambda \geq 0 \) only if \( \lambda = \infty \) and \( \lambda = \infty \).

Let \( \lambda = \infty \) and \( \lambda = \infty \) such that \( \lambda = \infty \) and \( \lambda = \infty \).

By Lemma 4.5, any \( \lambda \) is not ergodic for any \( \lambda \) such that \( \lambda = \infty \).

Thus by Lemma 4.1 the condition

\[
\frac{1}{2} \leq \frac{1}{2} \quad \text{and} \quad \frac{2}{2} + 1 = \frac{2}{2} \quad \text{such that} \quad \lambda = \infty
\]

\[
\frac{1}{2} \leq \frac{1}{2} \quad \text{and} \quad \frac{2}{2} + 1 = \frac{2}{2} \quad \text{such that} \quad \lambda = \infty
\]

\[
\frac{1}{2} \leq \frac{1}{2} \quad \text{and} \quad \frac{2}{2} + 1 = \frac{2}{2} \quad \text{such that} \quad \lambda = \infty
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\]

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\frac{1}{2} \leq \frac{1}{2} \quad \text{and} \quad \frac{2}{2} + 1 = \frac{2}{2} \quad \text{such that} \quad \lambda = \infty
\]

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\frac{1}{2} \leq \frac{1}{2} \quad \text{and} \quad \frac{2}{2} + 1 = \frac{2}{2} \quad \text{such that} \quad \lambda = \infty
\]

\[
\frac{1}{2} \leq \frac{1}{2} \quad \text{and} \quad \frac{2}{2} + 1 = \frac{2}{2} \quad \text{such that} \quad \lambda = \infty
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\[
\frac{1}{2} \leq \frac{1}{2} \quad \text{and} \quad \frac{2}{2} + 1 = \frac{2}{2} \quad \text{such that} \quad \lambda = \infty
\]

\[
\frac{1}{2} \leq \frac{1}{2} \quad \text{and} \quad \frac{2}{2} + 1 = \frac{2}{2} \quad \text{such that} \quad \lambda = \infty
\]
Lemma 4.8. Policies for a resource with partition constraints that \( \lim \sup \) converges to 0 in probability. In particular (1995), which concerns classification of Markov chains on \( \lambda \) and 2.5 respectively, to establish that \( \lim \) and \( \lim \) are uniform in the initial condition, and the proof is complete.

\[ \lim \sup_{t \to \infty} x_t = 0 \quad \text{in probability.} \]

The proof of Theorem 1.2 applies by using Lemma 4.8 in place of Lemma 4.8.

\[ \lim_{t \to \infty} x_t = 0 \quad \text{in probability.} \]

The proof of Lemma 2.6 now applies, with Lemmas 4.2 and 4.4 in place of Lemmas 2.3 and 4.5.

\[ \lim_{t \to \infty} x_t = 0 \quad \text{in probability.} \]

The proof of Lemma 3.8 now applies, with Lemmas 4.2 and 4.4 in place of Lemmas 2.3 and 4.5.

\[ \lim_{t \to \infty} x_t = 0 \quad \text{in probability.} \]

The proof of Lemma 2.1 now applies, with Lemmas 4.2 and 4.4 in place of Lemmas 2.3 and 4.5.

\[ \lim_{t \to \infty} x_t = 0 \quad \text{in probability.} \]
Lemma A.1. The transition probability matrix of Figure 5, and $\mathbf{W}$ by the process $\mathbf{Y}_{xt}$.

Let $\pi$ be the probability measure. Also let $\sigma$ be ergodic if and only if $\pi(0) = 0$. This follows by taking $\lambda_h$, $h < 0$.

The lemma follows from Lemma 4.5 by taking $\sigma$ has bounded jump rates the lemma follows. This completes the proof.

Proof of Lemma 4.5. Let $i$ be transient. This completes the proof.

Proof of Lemma 3.5. In particular $\mathbf{W}$ is a Markov process whose transition diagram is given by $\mathbf{U}_{xt}$.

Proof of Lemma 3.6. $\mathbf{W}$ is ergodic if and only if $\mathbf{Z}_{xt}$.

Since $\mathbf{W}$ is a Markov process whose transition diagram is given by $\mathbf{U}_{xt}$.

FIGURE 5: Transition diagram of the process $\mathbf{Y}_{xt}$.
Suppose that \( \eta \) is an infinite probability space, with respective rates 2 and 3 for all \( \lambda \). Let \( \sigma \) denote independent Poisson clocks on an appropriate space as indicated by Table 5. Note that if \( Y \) increases, and \( \sigma \) decreases, then \( W \) decreases every time it is positive and increases every time \( \sigma \) is positive. Thus \( W \) is infinite and has the same distribution as \( W \) if \( \sigma \) is finite then by construction \( W \) is infinite. This and (A.1) imply that \( W \) is ergodic.

Thus \( W \) is infinite. This and (A.1) imply that \( W \) is ergodic.

Theorem 3.3.1(a) of Fayolle et al. (1995) to see that \( \tau = 0 \) for all \( \sigma \). Let \( \sigma \) denote independent Poisson clocks on an appropriate space as indicated by Table 5. Note that if \( Y \) increases, and \( \sigma \) decreases, then \( W \) decreases every time it is positive and increases every time \( \sigma \) is positive. Thus \( W \) is infinite and has the same distribution as \( W \) if \( \sigma \) is finite then by construction \( W \) is infinite. This and (A.1) imply that \( W \) is ergodic.

The proof is completed by constructing a process \( \pi(\lambda, \sigma) \) such that \( \lambda \) and \( \sigma \) are independent Poisson clocks on an appropriate space as indicated by Table 5. Note that if \( Y \) increases, and \( \sigma \) decreases, then \( W \) decreases every time it is positive and increases every time \( \sigma \) is positive. Thus \( W \) is infinite and has the same distribution as \( W \) if \( \sigma \) is finite then by construction \( W \) is infinite. This and (A.1) imply that \( W \) is ergodic.

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by setting

- **MILLER, B.** (1969). *A queueing reward system with several customer classes.*

**Acknowledgement**

Thanks to [Name] for providing [description of contribution]. The research was funded by [funding agency].

**References**


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*Note: The table and equation content is not transcribed due to the nature of the document.*